CHAPTER 1

MATHEMATICS AND UNITS OF MEASUREMENT

Mathematics is the Engineering Aid's basic tool. The use of mathematics is found in every rating in the Navy, from the simple arithmetic of counting for inventory purposes to the complicated equations encountered in computer and engineering designs. In the Occupational Field 13 ratings, the Engineering Aid is looked upon as superior in knowledge when it comes to the subject of mathematics, which generally is a correct assumption; however, to be worthy of this calling, you have the responsibility to learn more about this subject. Mathematics is a broad science that cannot be covered fully in formal service school training, so it is up to you to devote some of your own time to the study of this subject.

The EA must have the ability to compute easily, quickly, systematically, and accurately. This requires a knowledge of the fundamental properties of numbers and the ability to estimate the accuracy of computations based on field measurements or collected field data. To compute rapidly, you need constant practice and should be able to use any available device to speed up and simplify computations. In solving a mathematical problem, you should take a different approach than you would if it were simply a puzzle you were solving for fun. Guesswork has no place in its consideration, and the statement of the problem itself should be devoid of anything that might obscure its true meaning. Mathematics is not a course in memory but one in reasoning. Mathematical problems should be read and so carefully analyzed that all conditions are well fixed in mind. Avoid all unnecessary work and shorten the solution wherever possible. Always apply some proof or check to your work. Accuracy is of the greatest importance; a wrong answer is valueless.

This chapter covers various principles of mathematics. The instructions given will aid the EA in making mathematical computations in the field and the office. This chapter also covers units of measurement and the conversion from one system to the other; that is, from the English to the metric system.

FUNDAMENTALS OF MATHEMATICS

MATHEMATICS is, by broad definition, the science that deals with the relationships between quantities and operations and with methods by which these relationships can be applied to determine unknown quantities from given or measured data. The fundamentals of mathematics remain the same, no matter to what field they are applied. Various authors have attempted to classify mathematics according to its use. It has been subdivided into a number of major branches. Those with which you will be principally concerned are arithmetic, algebra, geometry, and trigonometry.

ARITHMETIC is the art of computation by the use of positive real numbers. Starting with the review of arithmetic, you will, by diligent effort, build up to a study of algebra.

ALGEBRA is the branch of mathematics that deals with the relations and properties of numbers by means of letters, signs of operation, and other symbols. Algebra includes solution of equations, polynomials, verbal problems, graphs, and so on.

GEOMETRY is the branch of mathematics that investigates the relations, properties, and measurement of solids, surfaces, lines, and angles; it also deals with the theory of space and of figures in space.

TRIGONOMETRY is the branch of mathematics that deals with certain constant relationships that exist in triangles and with methods by which they are applied to compute unknown values from known values.

STUDY GUIDES

Mathematics is an exact science, and there are many books on the subject. These numerous books are the result of the mathematicians' efforts to solve mathematical problems with ease. Methods of arriving at solutions may differ, but the end results or answers are always the same. These different approaches to mathematical problems make the study of mathematics more interesting, either by individual study or as a group.

You can supplement your study of mathematics with the following training manuals:

1. *Mathematics, Vol. 1,* NAVEDTRA 10069-D1
2. *Mathematics, Vol. 2-A,* NAVEDTRA 10062
3. *Mathematics, Vol. 2-B,* NAVEDTRA 10063
4. *Mathematics, Vol. 3,* NAVEDTRA 10073-A1

TYPES OF NUMBERS

Positive and negative numbers belong to the class called REAL NUMBERS. Real numbers and imaginary numbers make up the number system in algebra. However, in this training manual, we will deal only with real numbers unless otherwise indicated.

A real number may be rational or irrational. The word *rational* comes from the word *ratio.* A number is rational if it can be expressed as the quotient, or ratio, of two whole numbers. Rational numbers include fractions like 2/7, whole numbers (integers), and radicals if the radical is removable. Any whole number is rational because it could be expressed as a quotient with 1 as its denominator. For instance, 8 equals 8/1, which is the quotient of two integers. A number like $\sqrt{16}$ is rational since it can be expressed as the quotient of the two integers in the form 4/1. An irrational number is a real number that cannot be expressed as the ratio of two integers. The numbers

$$\sqrt{3}, \quad 5\sqrt{2}, \quad \sqrt{7+5}, \quad \frac{3}{8}\sqrt{20}, \quad \frac{3}{\sqrt{5}}$$

and 3.1416 (π) are examples of irrational numbers.

An integer may be prime or composite. A number that has factors other than itself and 1 is a composite number. For example, the number 15 is composite. It has the factors 5 and 3. A number that has no factors except itself and 1 is a prime number. Since it is advantageous to separate a composite number into prime factors, it is helpful to be able to recognize a few prime numbers. The following are examples of prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, and 23.

A composite number may be a multiple of two or more numbers other than itself and 1, and it may contain two or more factors other than itself and 1. Multiples and factors of numbers are as follows: Any number that is exactly divisible by a given number is a multiple of the given number. For example, 24 is a multiple of 2, 3, 4, 6, 8, and 12 since it is divisible by each of these numbers. Saying that 24 is a multiple of 3, for instance, is equivalent to saying that 3 multiplied by some whole number will give 24. Any number is a multiple of itself and also of 1.

FRACTIONS, DECIMALS, AND PERCENTAGES

The most general definition of a fraction states that "a fraction is an indicated division. " Any division may be indicated by placing the dividend over the divisor with a line between them. By the above definition, any number, even a so-called "whole" number, may be written as a common fraction. The number 20, for example, may be written as 20/1. This or any other fraction that amounts to more than 1 is an IMPROPER fraction. For example, 8/3 is an improper fraction, The accepted practice is to reduce an improper fraction to a mixed fraction (a whole number plus a proper fraction). Perform the indicated division and write the fractional part of the quotient in its lowest term. In this case, 8/3 would be 2 2/3. A fraction that amounts to less than 1 is a PROPER fraction, such as the fraction 1/4.

To refresh your memory, we are including the following rules in the solution of fractions:

1. If you multiply or divide both the numerator and denominator of a fraction by the same number, the value does not change. The resulting fraction is called an EQUIVALENT fraction.
2. You can add or subtract fractions only if the denominators are alike.
3. To multiply fractions, simply find the products of the numerators and the products of the denominators. The resulting fractional product must be reduced to the lowest term possible.

4. To divide a fraction by a fraction, invert the divisor and proceed as in multiplication.

5. The method of CANCELING can be used to advantage before multiplying fractions (using the principle of rule No. 1) to avoid operations with larger numbers.

A decimal fraction is a fraction whose denominator is 10 or some power of 10, such as 100, 1,000, and so on. For example,

$$\frac{7}{10}, \frac{23}{100}, \text{ and } \frac{87}{1,000}$$

are decimal fractions. Accordingly, they could be written as 0.7, 0.23 and 0.087 respectively. Decimal fractions have certain characteristics that make them easier to use in computations than other fractions. Chapter 5 of NAVEDTRA 10069-D1 deals entirely with decimal fractions. A thorough understanding of decimals will be useful to the Engineering Aid in making various engineering computations. Figure 1-1 shows decimal equivalents of fractions commonly used by Builders, Steelworkers, Utilitiesmen, and other trades.

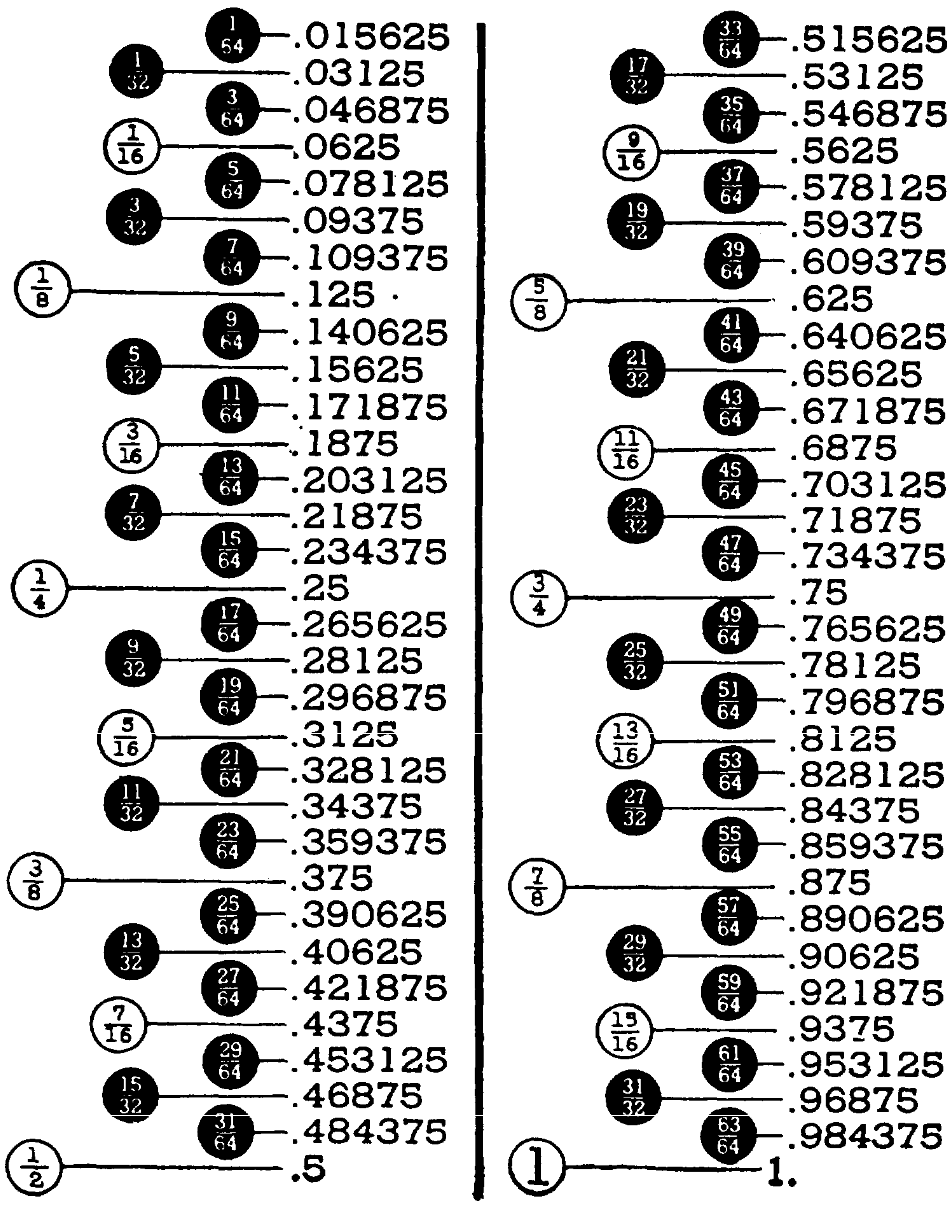

Figure 1-1. Decimal equivalents.

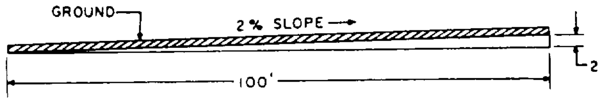

Figure 1-2.-2-percent grade.

In connection with the study of decimal fractions, businessmen as early as the fifteenth century made use of certain decimal fractions so much that they gave them the special designation PERCENT. The word *percent is* derived from Latin. It was originally *per centum*, which means "by the hundredths." In banking, interest rates are always expressed in percent; statisticians use percent; in fact, people in almost all walks of life use percent to indicate increases or decreases in production, population, cost of living, and so on. The Engineering Aid uses percent to express change in grade (slope), as shown in figure 1-2. Percent is also used in earthwork computations, progress reports, and other graphical representations. Study chapter 6 of NAVEDTRA 1-0069-D1 for a clear understanding of percentage.

POWERS, ROOTS, EXPONENTS, AND RADICALS

Any number is a higher power of a given root. To raise a number to a power means to multiply, using the number as a factor as many times as the power indicates. A particular power is indicated by a small numeral called the EXPONENT; for example, the small 2 on 3^2 is an exponent indicating the power.

Examples:

$$3^2 = 3 \times 3 = 9$$

$$3^3 = 3 \times 3 \times 3 = 27$$

$$6^2 = 6 \times 6 = 36$$

$$6^3 = 6 \times 6 \times 6 = 216$$

Many formulas require the power or roots of a number. When an exponent occurs, it must always be written unless its value is 1.

A particular ROOT is indicated by the radical sign ($\sqrt{\ }$), together with a small number called the INDEX of the root. The number under the radical sign is called the RADICAND. When the radical sign is used alone, it is generally understood to mean a square root, and $\sqrt[3]{\ }$, $\sqrt[5]{\ }$, and $\sqrt[7]{\ }$, indicate cube, fifth, and seventh roots, respectively. The square root of a number may be either + or – . The square root of 36 may be written thus: $\sqrt{36} = \pm 6$, since 36 could have been the product of (+ 6)(+ 6) or (– 6)(– 6). However, in practice, it is more convenient to disregard the double sign (±). This example is what we call the root of a perfect square. Sometimes it is easier to extract part of a root only after separation of the factors of the number, such as: $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$. As you can see, we were able to extract only the square root of 9, and 3 remains in the radical because it is an irrational factor. This simplification of the radical makes the solution easier because you will be dealing with perfect squares and smaller numbers.

Examples:

$$\sqrt{25} = \sqrt{5 \times 5} = 5$$

$$\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6} = 2 \times 2.236 = 4.472$$

$$\sqrt[3]{40} = \sqrt[3]{8 \times 5} = 2\sqrt[3]{5} = 2 \times 1.710 = 3.420$$

Radicals are multiplied or divided directly.

Examples:

$$\sqrt{3} \times \sqrt{6} = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$\frac{\sqrt{12}}{\sqrt{3}} = \frac{\sqrt{4} \times \sqrt{3}}{\sqrt{3}} = \sqrt{4} = \pm 2$$

Like fractions, radicals can be added or subtracted only if they are similar.

Examples:

$$2\sqrt{5} + \sqrt{5} = 3\sqrt{5}$$

$$\sqrt{2 \times 4} + \sqrt{2 \times 9} = \sqrt{2}(\sqrt{4}) + \sqrt{2}(\sqrt{9})$$

$$= 2\sqrt{2} + 3\sqrt{2}$$

$$= 5\sqrt{2}$$

When you encounter a fraction under the radical, you have to RATIONALIZE the denominator before performing the indicated operation. If you multiply the numerator and denominator by the same number, you can extract the denominator, as indicated by the following example:

$$\sqrt{\frac{2}{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{\sqrt{25}} = \frac{1}{5}\sqrt{10}$$

The same is true in the division of radicals; for example,

$$\sqrt{\frac{3}{6}} = \frac{\sqrt{3}}{\sqrt{6}} \div \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{2}}$$

Any radical expression has a decimal equivalent, which may be exact if the radicand is a rational number. If the radicand is not rational, the root may be expressed as a decimal approximation, but it can never be exact. A procedure similar to long division may be used for calculating square root. Cube root and higher roots may be calculated by methods based on logarithms and higher mathematics. Tables of powers and roots have been calculated for use in those scientific fields in which it is frequently necessary to work with roots. Such tables may be found in appendix I of *Mathematics, Vol. 1,* NAVEDTRA 10069-D 1, and in *Surveying Tables and Graphs,* Army TM 5-236. This method is, however, slowly being phased out and being replaced by the use of hand-held scientific calculators.

Arithmetic Extraction of Square Roots

If you do not have an electronic calculator, you may extract square roots arithmetically as follows:

Suppose you want to extract the square root of 2,034.01. First, divide the number into two-digit groups, working away from the decimal point. Thus set off, the number appears as follows:

$$\sqrt{20\ 34.01}$$

Next, find the largest number whose square can be contained in the first group, This is the number 4, whose square is 16. The 4 is the first digit of your answer. Place the 4 above the 20, and place its square (16) under the first group, thus:

$$\begin{array}{l} \quad 4 \\ \sqrt{20\ 34.01} \\ \quad \underline{16} \end{array}$$

Now perform the indicated subtraction and bring down the next group to the right, thus:

$$\begin{array}{l} \quad 4 \\ \sqrt{20\ 34.01} \\ \quad \underline{16} \\ \quad\ \ 434 \end{array}$$

Next, double the portion of the answer already found (4, which doubled is 8), and set the result down as the first digit of a new divisor, thus:

$$\begin{array}{l} \quad 4 \\ \sqrt{20\ 34.01} \\ \quad \underline{16} \\ 8\ /434 \end{array}$$

The second digit of the new divisor is obtained by a trial-and-error method. Divide the single digit 8 into the first two digits of the remainder 434 (that is, into 43) until you obtain the largest number that you can (1) add as another digit to the divisor and (2) use as a multiplier which, when multiplied by the increased divisor, will produce the largest result containable in the remainder 434. In this case, the first number you try is 43 + 8, or 5. Write this 5 after the 8 and you get 85. Multiply 85 by 5 and you get 425, which is containable in 434.

The second digit of your answer is therefore 5. Place the 5 above 34. Your computation will now look like this:

$$\begin{array}{l} \quad 4\ \ 5 \\ \sqrt{20\ 34.01} \\ \quad \underline{16} \\ 85/434 \\ \quad\ \ \underline{425} \end{array}$$

Proceed as before to perform the indicated subtraction and bring down the next group, thus:

$$\begin{array}{l} \quad 4\ \ 5 \\ \sqrt{20\ 34.01} \\ \quad \underline{16} \\ 85/434 \\ \quad\ \ \underline{425} \\ \qquad\ \ 901 \end{array}$$

Again double the portion of the answer already found, and set the result (45 x 2, or 90) down as the first two digits of a new divisor thus:

$$\begin{array}{l} \quad 4\ \ 5 \\ \sqrt{20\ 34.01} \\ \quad \underline{16} \\ 85/434 \\ \quad\ \ \underline{425} \\ 90/\quad 901 \end{array}$$

Proceed as before to determine the largest number that can be added as a digit to the divisor 90 and used as a multiplier which, when multiplied by the increased divisor, will produce a result containable in the remainder, 901. This number is obviously 1. The increased divisor is 901, and this figure, multiplied by the 1, gives a result exactly equal to the remainder 901.

The figure 1 is therefore the third and final digit in the answer, The square root of 2,034.01 is therefore 45.1

Your completed computation appears thus:

```
       4   5. 1
    √ 20  34.01
      16
    85/434
       425
    901/ 901
         901
```

Fractional and Negative Exponents

In some formulas, like the velocity (V) of liquids in pipes, which you will encounter later in *Engineering Aid 1 &* C, it is more convenient to use FRACTIONAL EXPONENTS instead of radicals.

Examples:

$$\sqrt{3} = 3^{\frac{1}{2}}$$

$$\sqrt[3]{3} = 3^{\frac{1}{3}}$$

$$\sqrt[3]{3^2} = 3^{\frac{2}{3}}$$

It is readily observed that the index of the root in the above examples is the denominator of the fractional exponent. When an exponent occurs in the radicand, this exponent becomes the numerator of the fractional exponent. Roots of numbers not found in tables may be easily computed by proper treatment of the radical used.

Examples:

$$\sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{\sqrt{16}} = \frac{1}{4}\sqrt{7} = \frac{2.646}{4} = 0.06615$$

$$\sqrt{8\frac{3}{4}} = \sqrt{\frac{35}{4}} = \frac{\sqrt{35}}{\sqrt{4}} = \frac{1}{2}\sqrt{35} = \frac{5.916}{2} = 2.958$$

In some work, NEGATIVE exponents are used instead of the reciprocals of numbers.

Examples:

$$3^{-1} = \frac{1}{3} \qquad 10^{-1} = \frac{1}{10}$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9} \qquad 10^{-2} = \frac{1}{100}$$

$$\frac{1}{5^{-1}} = 5 \qquad 10^{-3} = \frac{1}{1{,}000}$$

Very small or very large numbers used in science are expressed in the form 5.832×10^{-4} or 8.143×10^{6} to simplify computation. To write out any of these numbers in full, just move the decimal point to either left or right, the number of places equal to the exponent, supplying a sufficient number of zeros depending upon the sign of the exponent, as shown below:

$5.832 \times 10^{-4} = 0.0005832$ *(decimal moved four places to the left)*

$8.143 \times 10^{6} = 8{,}143{,}000$ (decimal moved six places to the right)

RECIPROCALS

The reciprocal of a number is 1 divided by the number. The reciprocal of 2, for example, is 1/2, and the reciprocal of 2/3 is 1 divided by 2/3, which amounts to 1 x 3/2, or 3/2. The reciprocal of a whole number, then, equals 1 over the number, while the reciprocal of a fraction equals the fraction inverted.

In problems containing the power of 10, generally, it is more convenient to use reciprocals rather than write out lengthy decimals or whole numbers.

Example:

$$\frac{1}{250{,}000 \times 300 \times 0.02} = \frac{1}{2.5 \times 10^5 \times 3 \times \cancel{10^2} \times 2 \times \cancel{10^{-2}}}$$

$$= \frac{10^{-5}}{2.5 \times 3 \times 2} = \frac{10^{-5}}{15} = \frac{1 \times 10^{-5}}{15}$$

$$= .0667 \times 10^{-5} = 6.67 \times 10^{-2} \times 10^{-5}$$

$$= 6.67 \times 10^{-7}$$

$$= 0.000000667$$

Reciprocal is also used in problems involving trigonometric functions of angles, as you will see later in this chapter, in the solutions of problems containing identities.

RATIO AND PROPORTION

Almost every computation you will make as an EA that involves determining an unknown value from given or measured values will involve the solution of a proportional equation. A thorough understanding of ratio and proportion will greatly help you in the solution of both surveying and drafting problems.

The results of observation or measurement often must be compared to some standard value in order to have any meaning. For example, if the magnifying power of your telescope is 20 diameters and you see a telescope in the market that says 50 diameter magnifying power, then one can see that the latter has a greater magnifying power. How much more powerful? To find out, we will divide the second by the first number, which is

$$\frac{50}{20} = \frac{5}{2}.$$

The magnifying power of the second telescope is 2 1/2 times as powerful as the first. When the relationship between two numbers is shown this way, the numbers are compared as a RATIO. In mathematics, a ratio is a comparison of two quantities. Comparison by means of a ratio is limited to quantities of the same kind, For example, in order to express the ratio between 12 ft and 3 yd, both quantities must be written in terms of the same unit. Thus, the proper form of this ratio is 4 yd:3 yd, not 12 ft:3 yd. When the parts of the ratio are expressed in terms of the same unit, the units cancel each other and the ratio consists simply of two numbers. In this example, the final form of the ratio is 4:3.

Since a ratio is also a fraction, all the rules that govern fractions may be used in working with ratios. Thus, the terms may be reduced, increased, simplified, and so forth, according to the rules for fractions.

Closely allied with the study of ratio is the subject of proportion. A PROPORTION is nothing more than an equation in which the members are ratios. In other words, when two ratios are set equal to each other, a proportion is formed. The proportion may be written in three different ways, as in the following examples:

$$15:20::3:4:$$

$$15:20 = 3:4$$

$$\frac{15}{20} = \frac{3}{4}$$

The last two forms are the most common. All of these forms are read, "15 is to 20 as 3 is to 4." In other words, 15 has the same ratio to 20 as 3 has to 4.

The whole of chapter 13, NAVEDTRA 10069-D1, is devoted to an explanation of ratio and proportion, the solution of proportional equations, and the closely related subject of variation. In addition to gaining this knowledge, you should develop the ability to recognize a computational situation as one that is available to solution by proportional equation. A very large area of surveying computations—the area that involves triangle solutions—uses the proportional equation as the principal key to the determination of unknown values on the basis of known values. Practically any problem involving the conversion of measurement expressed in one unit to the equivalent in a different unit is solvable by proportional equation. Similarly, if you know the quantity of a certain material required to produce a certain number of units of product, you can determine by proportional equation the quantity required to produce any given number of units.

In short, it is difficult to imagine any mathematical computation involving the determination of unknown values on the basis of known values that is not available to solution by proportional equation.

Your knowledge of equations need not extend beyond that required to solve linear equations; that is, equations in which the unknown appears with no exponent higher than 1. The equation

$$4x + 7 = \frac{15}{6},$$

for example, is a linear equation, because the unknown (technically known as the "variable' '), x, appears to only the first power. The equation $X^2 + 2x = -1$, however, is a quadratic, not a linear, equation because the variable appears to the second power.

The whole of chapter 11 of NAVEDTRA 10069-D1 is devoted to an explanation of linear equations in one variable. The whole of chapter 12 is devoted to an explanation of linear equations in two variables.

ARITHMETIC

The common arithmetical operations are addition, subtraction, multiplication, and division. Arithmetical operations with positive whole numbers are explained in chapter 2 of NAVEDTRA 10069-D1, and arithmetical operations with signed numbers, in chapter 3. Arithmetical operations with common fractions are explained in chapter 4, and arithmetical operations with decimal fractions, in chapter 5.

ALGEBRAIC NOTATION AND ALGEBRAIC OPERATIONS

Algebraic notation—meaning generally the substitution of symbols (usually letters) for numerical values-is explained in chapter 9 of NAVEDTRA 10069-D1. Algebraic fundamentals, such as the meanings of terms; systems of groupings; and the addition, subtraction, multiplication, and division of algebraic monomials and polynomials are explained in the same chapter. The factoring of algebraic expressions is explained in chapter 10.

GEOMETRY

Since geometry is the branch of mathematics that investigates the relations, properties, and measurement of solids, surfaces, lines, and angles, it follows that just about everything a surveyor does involves geometry in some way or other. Whenever you establish a point, chain a linear distance, measure a vertical distance, turn an angle, or determine an area or a volume, you are working with geometry.

To begin with, you must know how to recognize the common types of geometrical plane and solid figures and how to compute the areas of the plane figures and the volumes of the solids.

SURFACES AND FIGURES

There is a surface on this sheet of paper. A geometrical surface has length and breadth. It has no thickness. A surface may be either a plane surface or a curved surface. When this page is held perfectly level at every point, the surface is then a plane surface. When the page is rolled to resemble a tube, the plane surface becomes a curved surface.

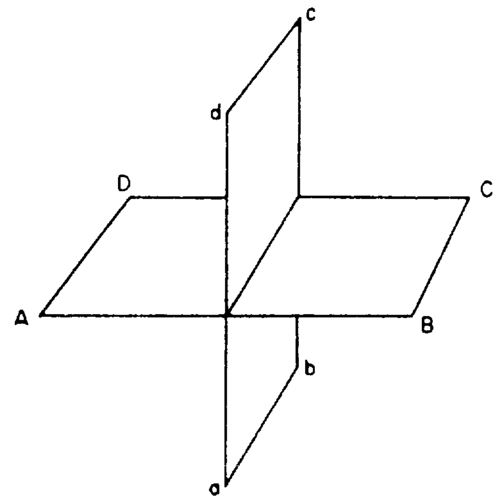

Figure 1-3.-Intersecting planes.

A plane is a real or imaginary surface in which a straight line between any two points lies wholly on that surface. Figure 1-3 shows two intersecting planes. Plane ABCD is shown to be a horizontal plane; plane abed is a vertical plane perpendicular to ABCD.

A plane surface is a surface on which every point lies in the same plane.

Plane figures are plane surfaces bounded by either straight lines or curved lines.

POLYGONS

A plane figure that is bounded by straight-line sides is called a polygon. The smallest possible number of sides for a polygon is three, and a three-sided polygon is called a triangle.

Some terms and definitions relating to polygons are as follows:

Sides	The boundary lines of a polygon
Perimeter	The sum of the sides

Triangle	A polygon bounded by three sides
Quadrilateral	A polygon bounded by four sides
Hexagon	A polygon bounded by six sides
Heptagon	A polygon bounded by seven sides
Octagon	A polygon bounded by eight sides
Equilateral	A polygon with sides of equal length
Regular	An equilateral polygon
Irregular	A nonequilateral polygon
Parallelogram	A quadrilateral with both pairs of opposite sides parallel
Rectangle	A parallelogram in which adjacent sides join at right angles
Square	An equilateral rectangle
Oblong	A nonequilateral rectangle
Trapezoid	A quadrilateral with only one pair of opposite sides parallel, the other pair being not parallel
Trapezium	A quadrilateral with no sides parallel
Rhombus	An equilateral parallelogram in which adjacent sides join at oblique (other than right) angles
Rhomboid	A nonequilateral parallelogram in which adjacent sides join at oblique angles

A triangle, quadrilateral, pentagon, hexagon, heptagon, and octagon are shown in figure 1-4. A trapezoid, trapezium, rhombus, and rhomboid are shown in figure 1-5.

DETERMINING AREAS

The area of any surface is the number of units of area measure the surface contains. A unit of area measure is a square unit. The main thing to remember when computing for areas is that the dimensions used must be of the same unit of measure—if in inches, all units must be in inches and if in feet, all must be in feet.

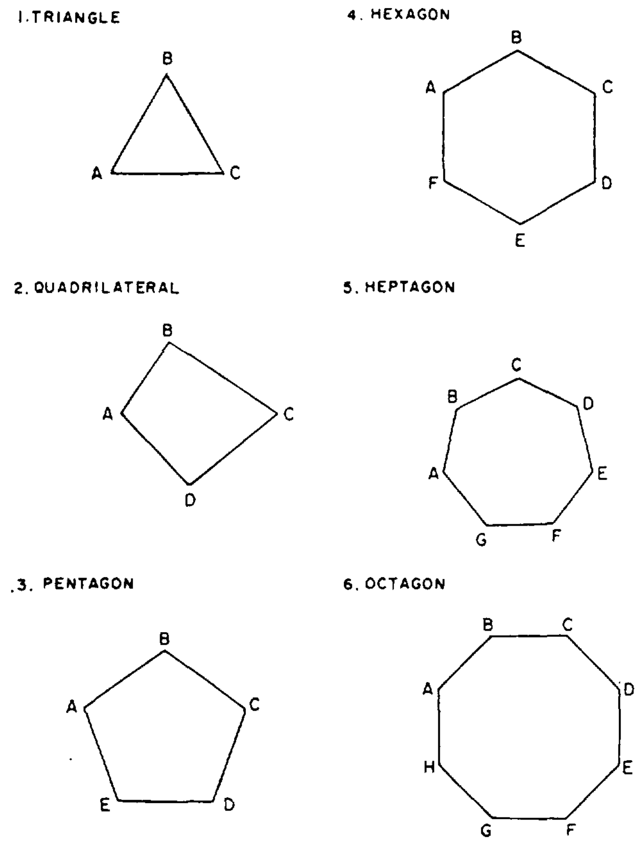

Figure 1-4.-Geometric figures of a triangle, quadrilateral, pentagon, hexagon, heptagon, and octagon.

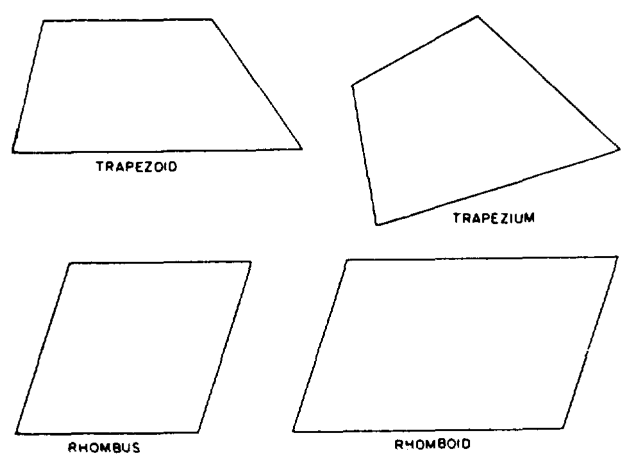

Figure 1-5.-Geometric figures of a trapezoid, trapezium, rhombus, and rhomboid.

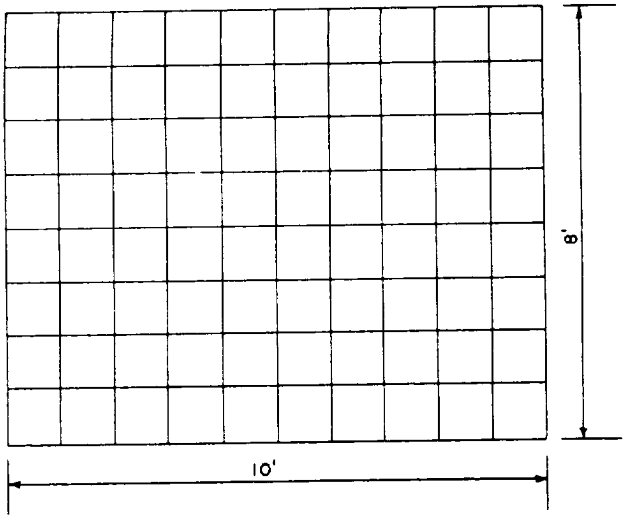

Figure 1-6.-Area of a rectangle.

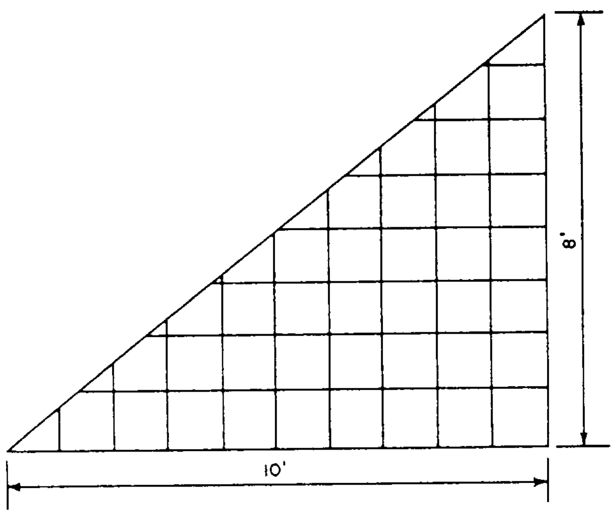

Figure 1-7.-Area of a triangle.

Area of a Rectangle

Figure 1-6 shows a rectangle measuring 10 ft by 8 ft, divided up into units of area measure, each consisting of 1 sq ft. If you were to count the units, one after the other, you would count a total of 80 units. However, you can see that there are 8 rows of 10 units, or 10 rows of 8 units. Therefore, the quickest way to count the units is simply to multiply 10 by 8, or 8 by 10.

You could call the 8-ft dimension the width and the 10-ft dimension the length, in which case you would say that the formula for determining the area of a rectangle is the width times the length, or A = w1. Or, you could call the 10-ft dimension the base and the 8-ft dimension the altitude (meaning height), in which case your formula for area of a rectangle would be A = bh.

Area of a Triangle

Figure 1-7 shows a triangle consisting of one-half of the rectangle shown in figure 1-6. It is obvious that the area of this triangle must equal one-half of the area of the corresponding rectangle, and the fact that it does can be demonstrated by geometrical proof. Therefore, since the formula for the area of the rectangle is A = bh, it follows that the formula for the triangle is A = 1/2bh.

The triangle shown in figure 1-7, because it is half of a corresponding rectangle, contains a right angle, and is therefore called a right triangle. In a right triangle the dimension h corresponds to the length of one of the sides. The triangle shown in figure 1-8, however, is a scalene triangle, so-called because no two sides are equal. Classification of triangles will be discussed later in this chapter.

Now, a perpendicular CD drawn from the apex of the triangle (from angle C) divides the triangle into two right triangles, ΔADC and ΔBDC. The area of the whole triangle equals the sum of the areas of ΔADC and ΔBDC. The area of ΔADC equals 1/2 (AD)(DC), and the area of ΔBDC equals 1/2(DB)(DC). Therefore, the area of the whole triangle equals

$$\frac{AD}{2}(DC) + \frac{DB}{2}(DC), \text{ or } DC\left(\frac{AD + DB}{2}\right).$$

But since AD + DB = AB, it follows that the area of the whole triangle equals

$$DC\left(\frac{AB}{2}\right).$$

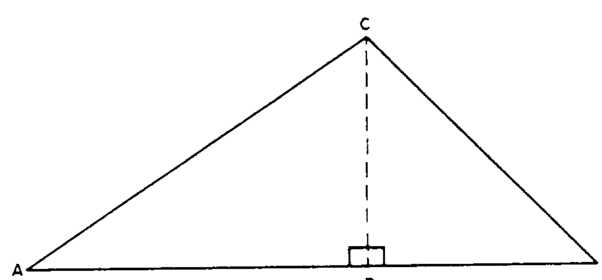

Figure 1-8.-Triangle.

The length of AB is called the base (b), and the length of DC, the altitude (h); therefore, your formula for determining the area of an oblique triangle is again A = 1/2bh.

You must remember that in a right triangle h corresponds to the length of one of the sides, while in an oblique triangle it does not. Therefore, for a right triangle with the length of the sides given, you can determine the area by the formula A = 1/2bh. For an oblique triangle with the length of the sides given, you cannot use this formula unless you can determine the value of h, Later in this chapter you will learn trigonometric methods of determining areas of various forms of triangles on the basis of the length of the sides alone.

Area of a Rhombus or Rhomboid

Figure 1-9 shows a rhomboid, ABCD. If you drop a perpendicular, CF, from ∠ C to AD, and project another from ∠A to BC, you will create two right triangles, ΔAEB and ΔCFD, and the rectangle AECF. It can be shown geometrically that the right triangles are similar and equal.

You can see that the area of the rectangle AECF equals the product of AF x FC. The area of the triangle CFD equals 1 /2(FD)(FC). Because the triangle AEB is equal and similar to CFD, the area of that triangle also equals 1/2(FD)(FC). Therefore, the total area of both triangles equals (FD)(FC). The total area of the rhomboid equals the area of the rectangle AECF + the total area of both triangles.

The total area of the rhomboid equals (AF)(FC) + (FD)(FC), or (AF + FD)(FC). But AF + FD equals AD, the base. FC equals the altitude. Therefore, the formula for the area of a rhomboid is A = bh. Here again you must remember that h in a rectangle corresponds to the length of one of the sides, but h in a rhombus or rhomboid does not.

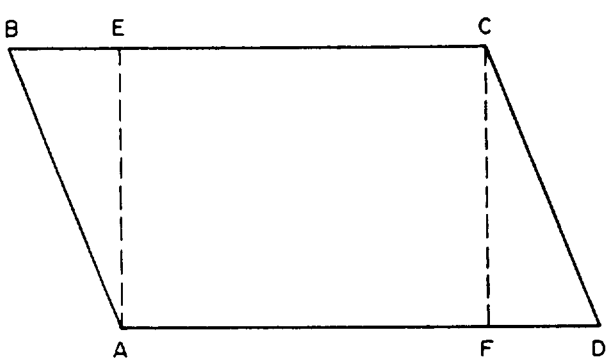

Figure 1-9.-Rhomboid.

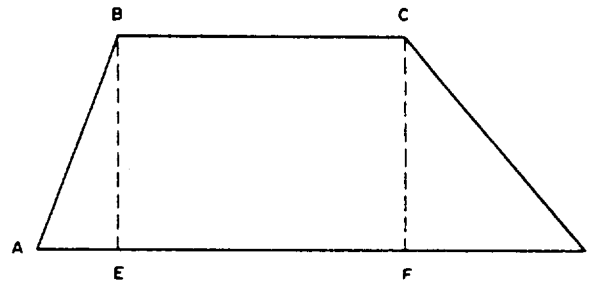

Figure 1-10.-Trapezoid.

Area of a Trapezoid

Figure 1-10 shows a trapezoid, ABCD. If you drop perpendiculars BE and CF from points B and C, respectively, you create the right triangles AEB and DFC and the rectangle EBCF between them. The area of the trapezoid obviously equals the sum of the areas of these figures.

The area of ΔAEB equals 1/2(AE)(FC), the area of ΔDFC equals 1/2(FD)(FC), and the area of EBCF equals (EF)(FC). Therefore, the area of the trapezoid ABCD equals l/2(AE)(FC) + (EF)(FC) + 1/2(FD)(FC), or

$$\frac{(AE + FD + 2EF)(FC)}{2}.$$

However, 2EF = EF + BC. Therefore, the area of the trapezoid equals

$$\frac{(AE + FD + EF + BC)(FC)}{2}.$$

But AE + FD + EF = AD. Therefore, the area of the trapezoid equals

$$\frac{(AD + BC)(FC)}{2}.$$

AD and BC are the bases of the trapezoid and are usually designated as b_1 and b_2, respectively. FC is the altitude and is generally designated as h. Therefore, the formula for the area of a trapezoid is

$$A = 1/2\ (b_1 + b_2)h.$$

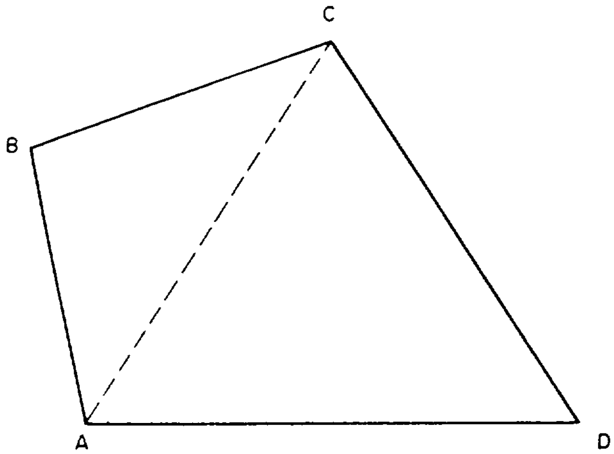

Figure 1-11.-Trapezium.

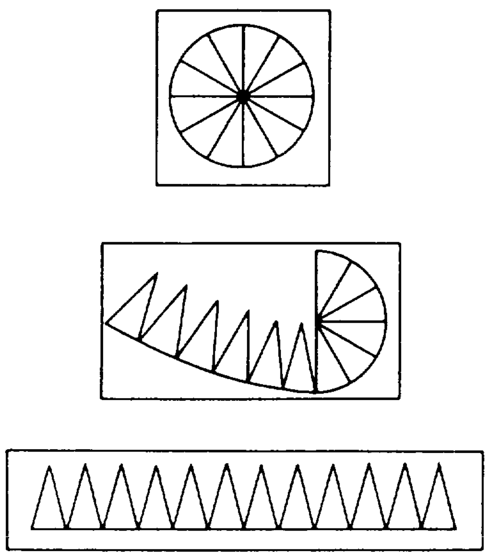

Figure 1-12.-Area of a circle.

Stated in words, the area of a trapezoid is equal to one-half the sum of its bases times its altitude.

Area by Reducing to Triangles

Figure 1-11 shows you how you can determine the area of a trapezium, or of any polygon, by reducing to triangles. The dotted line connecting A and C divides the figure into the triangles ABC and ACD. The area of the trapezium obviously equals the sum of the areas of these triangles.

Area of a Circle

Figure 1-12 shows how you could cut a disk into 12 equal sectors. Each of these sectors would constitute a triangle, except for the slight curvature of the side that was originally a segment of the circumference of the disk. If this side is considered the base, then the altitude for each triangle equals the radius (r) of the original disk. The area of each triangle, then, equals

$$\frac{br}{2},$$

and the area of the original disk equals the sum of the areas of all the triangles. The sum of the areas of all the triangles, however, equals the sum of all the b's, multiplied by r and divided by 2.

But the sum of all the b's equals the circumference (c) of the original disk. Therefore, the formula for the area of a circle can be expressed as

$$A = \frac{cr}{2}.$$

However, the circumference of a circle equals the product of the diameter times π (Greek letter, pronounced "pi"). π is equal to 3.14159. . . The diameter equals twice the radius; therefore, the circumference equals $2\pi r$. Substituting $2\pi r$ for c in the formula

$$A = \frac{cr}{2}, \text{ we have } A = \frac{(2\pi r)(r)}{2}, \text{ or } \frac{2\pi r^2}{2}, \text{ or } \pi r^2.$$

This is the most commonly used formula for the area of a circle. If we find the area of the circle in terms of circumference.

$$A = \frac{c^2}{4\pi}.$$

Area of a Segment and a Sector

A segment is a part of a circle bounded by a chord and its arc, as shown in figure 1-13. The formula for its area is

$$A = \frac{r^2}{2}\left(\frac{\pi n}{180} - \sin n\right)$$

where r = the radius and n = the central angle in degrees.

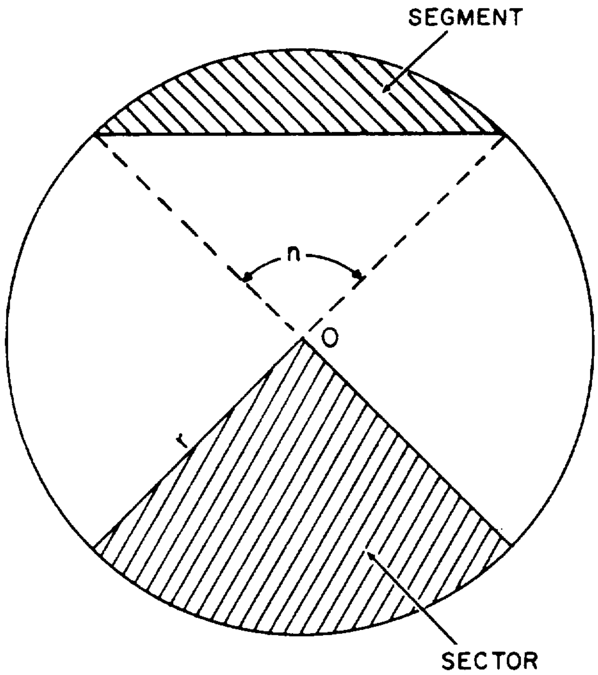

Figure 1-13.-Segment and sector of a circle,

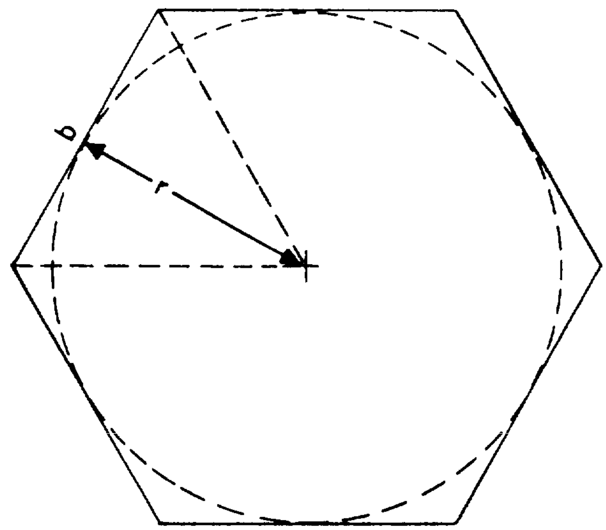

Figure 1-14.-Regular polygon.

A sector is a part of a circle bounded by two radii and their intercepted arc. The formula for its area is

$$A = \frac{\pi r^2 n}{360}$$

where r and n have the same designation as above.

Area of Regular Polygons

Figure 1-14 is a regular polygon. In any regular polygon, the area is equal to one-half the perimeter of the polygon times the radius of the inscribed circle. This is expressed in formula form as follows:

$$A = \frac{\text{perimeter} \times r}{2}$$

You can verify the above formula by dividing the polygon into equal triangles with the sides as their bases and with r as their altitudes; if you multiply the areas of the individual triangles by the number of sides in the polygon, you will arrive at the above formula.

Area of an Ellipse

The derivation of an ellipse from a conic section and methods of drawing ellipses are explained in chapter 3. An ellipse is shown in figure 1-15, The longer axis, AB, is called the major axis, and the shorter axis, CD, the minor axis. Call the length of the major axis a and that of the minor axis b. The area equals the product of half the major axis times half the minor axis times π. In formula form, it is stated as

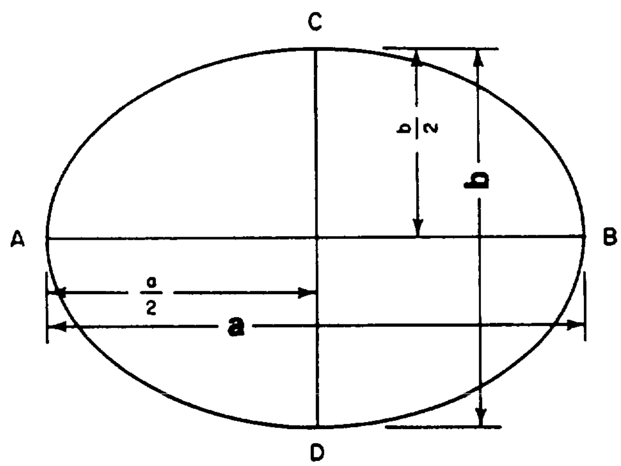

Figure 1-15.-Ellipse.

$$A = \pi\left(\frac{a}{2} \times \frac{b}{2}\right)$$

$$= \pi\left(\frac{ab}{4}\right)$$

$$= 0.7854ab$$

Irregular Areas

Irregular areas are those areas that do not fall within a definite standard shape. As you already have learned, there are formulas for computing the area of a circle, a rectangle, a triangle, and so on. However, we do not have a standard formula for computing the area of an irregular shaped plane, unless we use higher mathematics (calculus), and integrate incremental areas using lower and upper limits that define the boundaries.

As an EA, however, most areas you will be concerned with are those you will meet in plane surveying. In most surveys, the computed area is the horizontal projection of the area rather than the actual surface of the land. The fieldwork in finding areas consists of a series of angular and linear measurements, defining the outline of whatever the shape is of the area concerned, and forming a closed traverse. The following office computation methods, which you will learn as you advance in rate, are:

1. Plotting the closed traverse to scale and measuring the enclosed area directly with a polar planimeter (used only where approximate results are required, or for checking purposes).

2. Subdividing the area into a series of triangles, and taking the summation of all the areas of these triangles.

3. Computing the area using the coordinates of the individual points of the traverse (called coordinate method).

4. Computing the area by means of the balanced latitude and departure, and calculated DOUBLE MERIDIAN DISTANCES of each course (called the DMD method).

5. Computing the area by counting squares; this method is nothing but just superimposing small squares plotted on a transparent paper having the same scale as the plotted traverse (or of known graphical ratio) and counting the number of squares within the traverse. The smaller the squares, the closer to the approximate area you will get.

6. Computing an irregular area bounded by a curve and perpendicular lines, as shown in figure 1-16. Here, you can use the TRAPEZOIDAL RULE. The figure is considered as being made up of a series of trapezoids, all of them having the same base and having common distances between offsets. The formula in computing the total area is as follows:

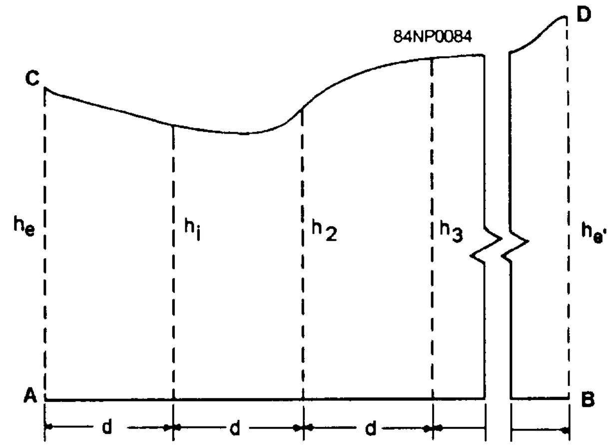

Figure 1-16.-Irregular area by trapezoidal rule.

$$A = \left(\frac{h_e}{2} + \sum h + h_{e'} \right) d$$

Where h_e and h_e = the end offsets of the series of trapezoids

$\sum h$ = the sum of the intermediate offsets ($h_1 + h_2 + h_3 + \ldots$)

and d = the common distance between the offsets

For the present time, try to find the areas of irregular figures by subdividing the area to series of triangles and by the method of counting the squares.

There are also areas of spherical surfaces and areas of portions of a sphere. For other figures not covered in this training manual, consult any text on plane and solid geometry.

DETERMINING VOLUMES

From the preceding section you learned the formulas for computing the areas of various plane figures. These plane areas are important in the computation of VOLUMES, as you will see later in this section.

When plane figures are combined to form a three-dimensional object, the resulting figure is

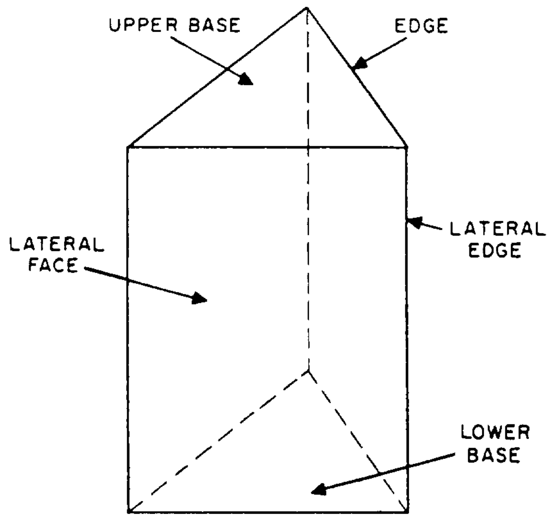

Figure 1-17.-Parts of a prism (triangular).

a solid. For example, three rectangles and two triangles may be combined as shown in figure 1-17. The flat surfaces of the solid figure are its FACES, the top and bottom faces are the BASES, and the faces forming the sides are the LATERAL FACES or SURFACES.

Some solid figures do not have any flat faces, and some have a combination of curved surfaces and flat surfaces. Examples of solids with curved surfaces include cylinders, cones, and spheres. Those solids having no flat faces include a great majority of natural objects, such as rocks, living matter, and many other objects that have irregular surfaces.

A solid figure whose bases or ends are similar, equal, and parallel polygons, and whose faces are parallelograms, is known geometrically as a PRISM. The name of a prism depends upon its base polygons. If the bases are triangles, as in figure 1-17, the figure is a TRIANGULAR PRISM. A RECTANGULAR PRISM has bases that are rectangles, as shown in figure 1-18. If the bases of a prism are perpendicular to the planes forming its lateral faces, the prism is a RIGHT prism.

A PARALLELEPIPED is a prism with parallelograms for bases. Since the bases are parallel to each other, this means that they cut the lateral faces to form parallelograms. If a parallelepipeds is a right prism and if its bases are rectangles, it is a rectangular solid. A CUBE is a rectangular solid in which all of the six rectangular faces are squares.

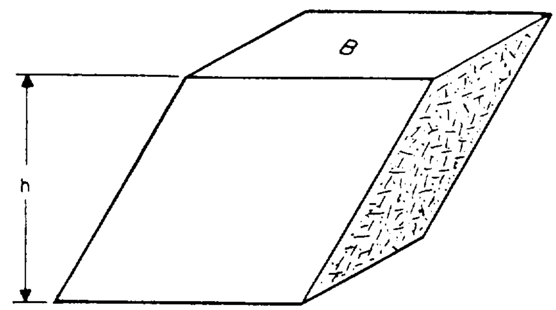

Figure 1-18.-Rectangular prism, showing its height when not a right prism.

In determining the volume of most solids, you should use the following general formula:

$$V = B\,h$$

Where V = the volume

B = the area of the base or end area

h = the height of the solid (the perpendicular height from its base)

Volume of a Prism

For the volume of any prism, then, you simply determine the end area or the base area by the appropriate method and multiply the end area by the length or the base area by the height.

Volume of a Cylinder

From the standpoint of volume calculation, the only difference between a cylinder and a prism lies in the fact that the end or base of a cylinder is a circle rather than a polygon. Therefore, the volume of a cylinder is equal to its end area times its length. But you determine its end area from the formula πr^2, which is the formula used for computing the area of a circular plane. Therefore, the volume of a cylinder is $\pi r^2 L$.

Volume of a Cone or Pyramid

The best way to approach the problem of determining the volume of a cone or pyramid is on the basis of the fundamental fact that the volume of a cone equals one-third of the volume of the corresponding cylinder, while the volume of a pyramid equals one-third of the volume of the corresponding prism. For any of these solids, volume equals base area times height divided by 3. Therefore, the formula for computing the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h,$$

and that for a pyramid is

$$V = \frac{1}{3}Bh.$$

A pyramid may have either a rectangular or a triangular base.

Volume of Other Geometric Figures

There will be no attempt to illustrate the derivation of formulas presented in this section. The formulas for the computations of volumes and surface areas of the following geometric figures are presented here only for additional information.

A frustum is that portion of a cone or pyramid that remains after the upper part is cut off by a plane parallel to the base.

1. SPHERE

   Volume of a sphere $= \frac{4}{3}\pi r^3$

   Surface area $= 4\pi r^2$

   Where r = the radius of the sphere

2. FRUSTUM OF A CONE

   Volume of frustum = volume of large cone − volume of small cone

   $= \frac{1}{3}\pi h(r_1{}^2 + r_1 r_2 + r_2{}'^2)$ cubic units

   Lateral area $= \pi(r_1 + r_2)s$ square units

   Where h = the altitude of the frustum

   r_1 = the radius of the base

   r_2 = the radius of the top

   s = the slant height

3. FRUSTUM OF A PYRAMID

   Volume of a frustum = volume of large pyramid − volume of small pyramid

   $= \frac{1}{3}h\,(B_1 + \sqrt{B_1 B_2} + B_2)$

   Where h = the altitude of frustum

   B_1 = the area of lower base

   B_2 = the area of upper base

TRIGONOMETRY

Our discussion will focus primarily on the study of plane trigonometry. It is intended only as a review of the relationships among the sides and angles of plane triangles and their ratios, called the TRIGONOMETRIC FUNCTIONS. The information presented here is based on *Mathematics, Vol. 1,* NAVEDTRA 10069-D1, chapter 19, and *Mathematics, Vol. 2-A,* NAVEDTRA 10062, chapters 3, 4, and 6.

Spherical trigonometry will be covered as you advance in rate. It is a prerequisite to the study of navigation, geodesy, and astronomy. Hence, the subject of spherical trigonometry will be introduced in the Engineering Aid class C1 school curriculum.

MEASURING ANGLES

When two straight lines intersect, an angle is formed, You can also generate an angle by rotating a line having a set direction, Figure 1-19 depicts the generation of an angle. The terminal line OB is generated from the initial point OA and forms $\angle$ AOB, which we will call θ (Greek letter, pronounced “theta”). Angle θ is generally expressed in degrees. The following paragraphs will discuss the degree and the radian systems that are generally used by Engineering Aids.

The DEGREE SYSTEM is the most common system used in angular measurement. Angular measurement by REVOLUTION is perhaps the unit you are most familiar with.

In the degree system, a complete revolution is divided into 360 equal parts called degrees (360°). Each degree is divided into 60 minutes (60´), and each minute into 60 seconds (60”). For convenience in trigonometric computations, the 360° is divided into four parts of 90° each. The

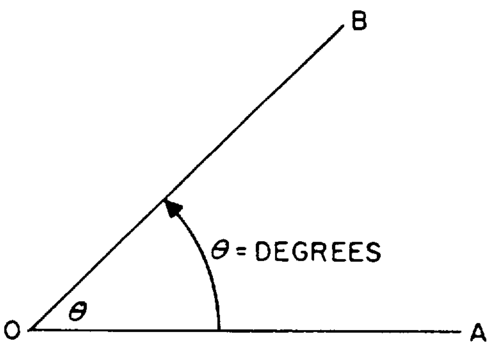

Figure 1-19.-Generation of an angle, resulting angle measured in degrees.

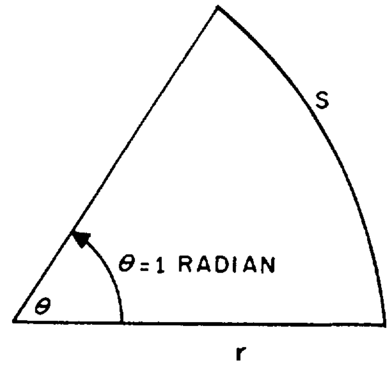

Figure 1-20.-Radian measure.

90° sectors, called QUADRANTS, are numbered counterclockwise starting at the upper right-hand sector.

When the unit radius r (the line generating the angle) has traveled less than 90° from its starting point in a counterclockwise direction (or, as conventionally referred to as, in a positive direction), the angle is in the FIRST quadrant (I). When the unit radius lies between 90° and 180°, the angle is in the SECOND quadrant (II). Angles between 180° and 270° are said to lie in the THIRD quadrant (III), and angles greater than 270° and less than 360° are in the FOURTH quadrant (IV).

When the line generating the angle passes through more than 360°, the quadrant in which the angle lies is found by subtracting from the angle the largest multiple of 360 that the angle contains and determining the quadrant in which the remainder falls.

The RADIAN SYSTEM of measuring angles is even more fundamental than the degree system. It has certain advantages over the degree system, for it relates the length of arc generated to the size of the angle and the radius. The radian measure is shown in figure 1-20. If the length of the arc (s) described by the extremity of the line segment generating the angle is equal to the length of the line (r), then it is said that the angle described is exactly equal to one radian in size; that is, for one radian, s = r.

The circumference of a circle is related to the radius by the formula, $C = 2\pi r$. This says that the circumference is 2π times the length of the radius. From the relationship of arc length, radius, and radians in the preceding paragraph, this could be extended to say that a circle

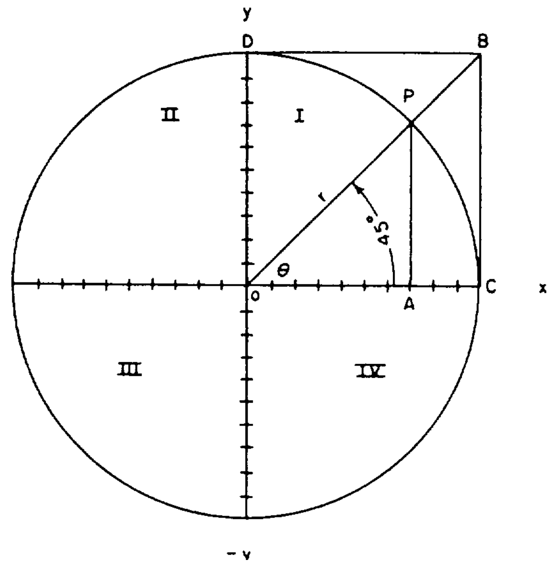

Figure 1-21.-Circle of unit radius with quadrants shown.

contains 2π radians, and the circumference encompasses 3600 of rotation. It follows that

$$2\pi \text{ radians} = 360°$$

$$\pi \text{ radians} = 180°$$

By dividing both sides of the above equation by π, we find that

$$\text{radian} = \frac{180°}{\pi} = 57.2959°, \text{ or } 57.3° \text{ (approximately)}$$

As in any other formula, you can always convert radians to degrees or vice versa by using the above relationship.

FUNCTIONS OF ANGLES

The functions of angles can best be illustrated by means of a "circle of unit radius" like the one shown in figure 1-21. A so-called "Cartesian axis" is inscribed within the circle. Coordinates measured from 0 along the x axis to thc right are positive; coordinates measured from 0 along the x axis to the left are negative. Coordinates measured along the y axis from 0 upward are positive; coordinates measured along the y axis from 0 downward are negative.

Angles are generated by the motion of a point P counterclockwise along the circumference of the circle. The initial leg of any angle is the positive leg of the x axis. The other leg is the radius r, at the end of which the point P is located; this radius always has a value of 1. The unit radius (r = OC) is subdivided into 10 equal parts, so the value of each of the 10 subdivisions shown is 0.1.

For any angle, the point P has three coordinates: the x coordinate, the y coordinate, and the r coordinate (which always has a value of 1 in this case). The functions of any angle are, collectively, various ratios that prevail between these coordinates.

The ratio between y and r (that is, y/r) is called the **sine** of an angle. In figure 1-21, AP seems to measure about 0.7 of y; therefore, the sine θ, which is equal to 45° in this case, would seem to be 0.7/1, or about 0.7. Actually, the sine of 45° is 0.70711. Graphically, the sine is indicated in figure 1-21 by the line AP, which measures 0.7 to the scale of the drawing.

The ratio between x and r (that is, x/r) is called the **cosine** of the angle. You can see that for 45°, x and y are equal, and the fact that they are can be proven geometrically. Therefore, the cosine of 45° is the same as the sine of 45°, or 0.70711. Graphically, the length of line OA represents the cosine of angle θ when the radius (r) is equal to 1.

The ratio between y and x (that is, y/x) is known as the **tangent** of an angle. Since y and x for an angle of 45° are equal, it follows that the tangent of an angle of 45° equals 1. The tangent is also indicated graphically by the line BC, drawn tangent to the circle at C and intersecting the extended r at B and DB, which is also drawn tangent at D. As you examine figure 1-21, you can deduce that BC is equal to OC. OC is equal to the unit radius, r.

The three functions shown in figure 1-21 are called the "direct" functions. For each direct function there is a corresponding "reciprocal" function–meaning a function that results when you divide 1 by the direct function. You know that the reciprocal of any fraction is simply the fraction inverted. Therefore, for the direct function sine, which is y/r, the reciprocal function (called the **cosecant)** is divided by y/r, which is r/y.

Since y at sine 45° equals about 0.7, the cosecant for 45° is r/y, which is equal to 1/0.7, or about 1.4. The cosecant is indicated graphically by the line OB in figure 1-21. If you measure this line, you will find that it measures just about 1.4 units to the scale of the drawing.

For the direct function cosine, which is x/r, the reciprocal function (called the **secant)** is r/x. Since x for cosine 45° also measures about 0.7, it follows that the secant for 45°, r/x, is the same as the cosecant, or also about 1.4. The secant is indicated graphically in figure 1-21 by the line OB also.

For the direct function tangent, which is y/x the reciprocal function (called the **cotangent)** is x/y. Since x and y at tangent 45° are equal, it follows that the value for cotangent 45° is the same as that for the tangent, or 1. The cotangent is shown graphically in figure 1-21 by the line DB, drawn tangent to the circle at D.

FUNCTIONS AND COFUNCTIONS

The functions cosine, cosecant, and cotangent are cofunctions of the functions sine, secant, and tangent, respectively. A cofunction of an angle A has the same value as the corresponding function of (90° – A); that is, the same value as the corresponding function of the complement of the angle. The sine of 30°, for example, is 0.50000. The cosine of 60° (the complement of 30°) is likewise 0.50000. The tangent of 30° is 0.57735. The cotangent of 60° (the complement of 30°) is likewise 0.57735.

Commonly used functions and cofunctions are as follows:

$$\sin A = \cos (90° - A)$$

$$\sec A = \csc (90° - A)$$

$$\tan A = \cot (90° - A)$$

FUNCTIONS OF OBTUSE ANGLES

In figure 1-22, the point P has generated an obtuse (larger than 90°) angle of 135°. This angle is the supplement of 45° (two angles are supplementary when they total 180°). We have left a dotted image of the reference angle A, which is equal to the supplementary angle of 135°. You

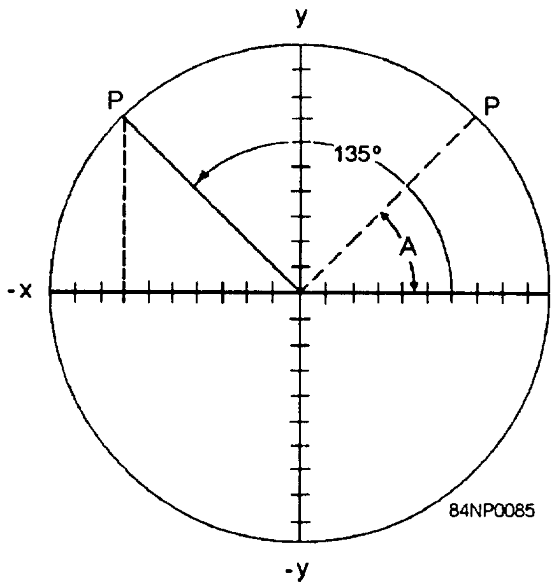

Figure 1-22.-Function of an obtuse angle.

can see that the values of x, y, and r are the same for 135° as they are for 45°, except that the value of x is negative. From this it follows that the functions of any obtuse angle are the same as the functions of its supplement, except that any function in which x appears has the opposite sign.

The sine of an angle is y/r. Since x does not appear in this function, it follows that sin A = sin (180° – A).

The cosine of an angle is fir. Since x appears in this function, it follows that cos A = - cos (180° – A).

The tangent of an angle is y/x. Since x appears in this function, it follows that tan A = – tan (180°- A).

The importance of knowing this lies in the fact that many tables of trigonometric functions list the functions only for angles to a maximum of 90°. Many oblique triangles, however, contain angles larger than 90°. To determine a function of an angle larger than 90° from a table that stops at 90°, you lookup the function of the supplement of the angle. If the function is a sine, you use it as is. If it is a cosine or tangent, you give it a negative sign.

The relationships of the function of obtuse angles are as follows:

$$\sin A = \sin (180° - A)$$

$$\cos A = -\cos (180° - A)$$

$$\tan A = -\tan (180° - A)$$

$$\cot A = -\cot (180° - A)$$

$$\sec A = -\sec (180° - A)$$

$$\csc A = \csc (180° - A)$$

The above relationships apply only when angle A is greater than 90° and less than 180°.

FUNCTIONS OF ANGLES IN A RIGHT TRIANGLE

For an acute angle in a right triangle, the length of the side opposite the angle corresponds to y and the length of the side adjacent to the angle corresponds to x, while the length of the hypotenuse corresponds to r. Therefore, the functions of an acute angle in a right triangle can be stated as follows:

$$\text{Sine} = \frac{\text{side opposition}}{\text{hypotenuse}} \qquad \text{Cosecant} = \frac{\text{hypotenuse}}{\text{side opposite}}$$

$$\text{Cosine} = \frac{\text{side adjacent}}{\text{hypotenuse}} \qquad \text{Secant} = \frac{\text{hypotenuse}}{\text{side adjacent}}$$

$$\text{Tangent} = \frac{\text{side opposite}}{\text{side adjacent}} \qquad \text{Cotangent} = \frac{\text{side adjacent}}{\text{side opposite}}$$

If you consider a 90° angle with respect to the “circle of unit radius” diagram, you will realize that for a 90° angle, x = 0, y = 1, and r (as always) equals 1. Since sine = y/r, it follows that the sine of 90° = 1. Since cosine = X/r, it follows that the cosine of 90° = 0/1, or 0. Since tangent= y/x, it follows that tan 90° = 1/0, or infinity (00). From one standpoint, division by 0 is a mathematical inpossibility, since it is impossible to state how many zeros there are in anything. From this standpoint, tan 90° is simply impossible. From another standpoint it can be said that there arc an “infinite” number of zeros in 1. From that standpoint, tan 90° can be said to be infinity.

In real life, the sides of a right triangle y, x, and r, or side opposite, side adjacent, and hypotenuse, are given other names according to the circumferences. In connection with a pitched roof rafter, for instance, y or side opposite is “total rise,” x or side adjacent is “total run,” and r or hypotenuse is “rafter length.” In connection with a ground slope, y or side opposite is “vertical rise,” x or side adjacent is “horizontal distance,” and r or hypotenuse is “slope distance.”

METHODS OF SOLVING TRIANGLES

To "solve" a triangle means to determine one or more unknown values (such as the length of a side or the size of an angle) from given known values. Here are some of the methods used.

Pythagorean Theorem

When you know the lengths of two sides of a right triangle, or its hypotenuse and one side, you can determine the length of the remaining side, or the length of the hypotenuse, by applying the Pythagorean theorem. The Pythagorean theorem states that the square of the length of the hypotenuse of any right triangle equals the sum of the squares of the lengths of the other two sides.

Figure 1-23 shows a right triangle with acute angles A and B and right angle C. Sides opposite A and B are designated as a and b; the hypotenuse (opposite C) is designated as c. Side a measures 3.00 ft, side b measures 4.00 ft, and the hypotenuse measures 5.00 ft. Any triangle with sides and hypotenuse in the ratio of 3:4:5 is a right triangle.

If $C^2 = a^2 + b^2$, it follows that $c = \sqrt{a^2 + b^2}$. The formulas for solving for either side, given the other side and the hypotenuse; or for the hypotenuse, given the two sides, are

$$a = \sqrt{c^2 - b^2}$$
$$b = \sqrt{c^2 - a^2}$$
$$c = \sqrt{a^2 + b^2}$$

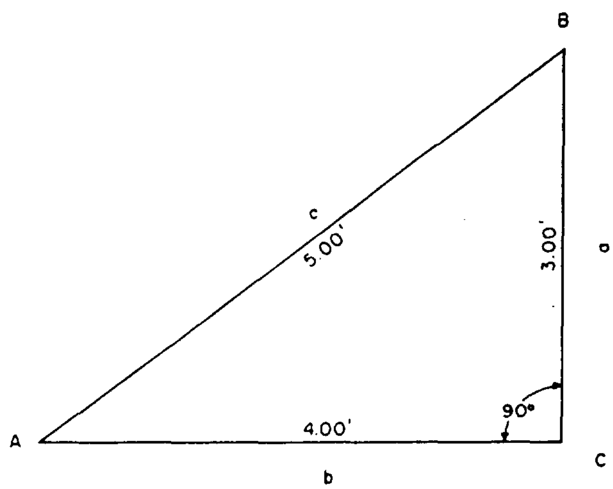

Figure 1-23.-A right triangle.

In figure 1-23, $a^2 = 9$, $b^2 = 16$, and $c^2 = 25$. Therefore, a = the square root of (25 − 16), or 3; b = the square root of (25 − 9), or 4; and c = the square root of (9 + 16), or 5.

Acute Angle of Right Triangle by Tangent

One of the angles in a right triangle always measures 900. Because the sum of the three angles in any triangle is always 180°, it follows that each of the other two angles in a right triangle must be an acute (less than 90°) angle. Also, if you know the size of one of the acute angles, you can determine the size of the other from the formulas A = (90° − B) and B = (90° − A).

In any right triangle in which you know the lengths of the sides, you can determine the size of either of the acute angles by applying the tangent of the angle. Take angle A in figure 1-23, for example. You know that

$$\tan A = \frac{a}{b}, \text{ or } \frac{3.00}{4.00}, \text{ or } 0.75.$$

Reference to a table of natural tangents shows that an angle with tangent 0.75 measures to the nearest minute, 36°52'.

Side of Right Triangle by Tangent

If you know the length of one of the sides of a right triangle and the size of one of the acute angles, you can determine the length of the other side by applying the tangent. Suppose that for the triangle shown in figure 1-23 you know that angle A measures 36°52' and that side b measures 4.00 ft. You want to determine the length of side a. Since

$$\tan A = \frac{a}{b},$$

it follows that a = b (tan A). From a table o natural tangents you find that tar 36°52′ = 0.74991. Therefore,

$$a = 4.00(0.74991), \text{ or } 3.00 \text{ ft.}$$

Side of Right Triangle by Cotangent

Suppose that for the triangle shown in figure 1-23, you know that angle B measures 53°08′ and that side a measures 3.00 ft. You want to determine the length of side b. You could do this as previously described by applying

$$\tan B = \frac{b}{a}.$$

However, the fact that side b is larger than side a means that tan B is larger than 1 (you recall that any angle larger than 45° has a tangent larger than 1).

You know that the cotangent is the reciprocal function of the tangent. Therefore, if

$$\tan B = \frac{b}{a}, \cot B = \frac{a}{b},$$

it follows that

$$b = \frac{a}{\cot B}.$$

A table of natural functions tells you that cot 53°08' = 0.74991. Therefore,

$$b = \frac{3}{0.74991}, \text{ or } 4.00.$$

Acute Angle of Right Triangle by Sine or Cosine

If you know the length of the hypotenuse and length of a side of a right triangle, you can determine the size of one of the acute angles by applying the sine or the cosine of the angle. Suppose that for the triangle shown in figure 1-23, you know that the hypotenuse, c, is 5.00 ft long and that the length of side a is 3.00 ft long. You want to determine the size of angle A. Side a is opposite angle A; therefore,

$$\sin A = \frac{a}{c}, \text{ or } \frac{3}{5}, \text{ or } 0.6.$$

A table of natural functions tells you that an angle with sine 0.6 measures (to the nearest minute) 36°52'.

Suppose that, instead of knowing the length of a, you know the length of b (4.00 ft). Side b is the side adjacent to angle A. You know that

$$\cos A = \frac{b}{c}, \text{ or } \frac{4}{5}, \text{ or } 0.8.$$

A table of natural functions tells you that an angle with cosine 0.8 measures 36°52'.

If you know the size of one of the acute angles in a right triangle and the length of the side opposite, you can determine the length of the hypotenuse from the sine of the angle. Suppose that for the triangle shown in figure 1-23, you know that angle A = 36°52' and side a = 3.00 ft.

$$\text{Sin } A = \frac{a}{c}; \text{ therefore, } c = \frac{a}{\sin A}, \text{ or } \frac{3}{0.6}, \text{ or } 5.00 \text{ ft.}$$

If you know the size of one of the acute angles in a right triangle and the length of the side adjacent, you can determine the length of the hypotenuse from the cosine of the angle. Suppose that for the triangle in figure 1-23, you know that angle A = 36°52' and side b = 4.00 ft.

$$\text{Cos } A = \frac{b}{c}; \text{ therefore, } c = \frac{b}{\cos A}.$$

Tables show that cos 36°52' = 0.80003. Therefore,

$$c = \frac{4.00}{0.80003}, \text{ or } 5.00 \text{ ft.}$$

Solution by Law of Sines

For any triangle (right or oblique), when you know the lengths of two sides and the size of the angle opposite one of them, or the sizes of two angles and the length of the side opposite one of them, you can solve the triangle by applying the law of sines. The law of sines (which is explained and proved in NAVPERS 10071-B, chapter 5) states that the lengths of the sides of any triangle are proportional to the sines of their opposite angles. It is expressed in formula form as follows:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

In the triangle shown in figure 1-24, $\angle A$ = 41°24', a = 8.00 ft, and b = 12.00 ft. If

$$\frac{b}{\sin B} = \frac{a}{\sin A},$$

it follows that

$$\sin B = \frac{b \sin A}{a}.$$

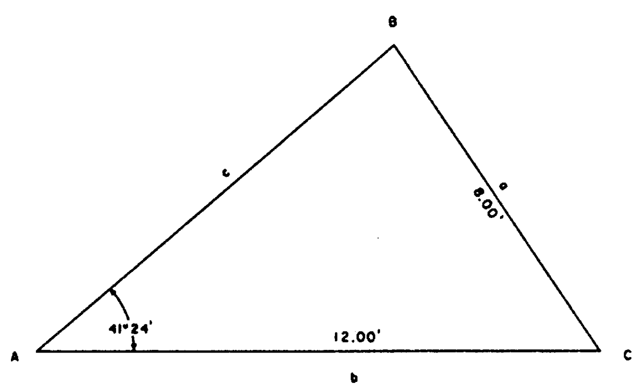

Figure 1-24.—Oblique triangle (law of sines).

The sine of 41°24′ is 0.66131; therefore,

$$\sin B = \frac{12(0.66131)}{8}, \text{ or } 0.99196.$$

Tables show that the angle with sine 0.99196 measures 82°44′. Therefore, $\angle B$ = 82°44′. $\angle C$ = 180° − (A + B), or 180° − (41°24′ + 82°44′), or 180° − 124°08, or 55°52′.

If $\frac{c}{\sin C} = \frac{a}{\sin A}$, then $c = \frac{a \sin C}{\sin A}$. The sine of 55°52′ is 0.82773. Therefore,

$$c = \frac{8(0.82773)}{9.66131}, \text{ or } 10.01 \text{ ft.}$$

Solution by Laws of Cosines

Suppose you know two sides of a triangle and the angle between the two sides. You cannot solve this triangle by the law of sines, since you do not know the length of the side opposite the known angle or the size of an angle opposite one of the known sides. In a case of this kind you must begin by solving for the third side by applying the law of cosines. The law of cosines is explained and proved in chapter 5 of NAVPERS 10071-B. If you are solving for a side on the basis of two known sides and the known included angle, the law of cosines states that in any triangle the square of one side is equal to the sum of the squares of the other two sides minus twice the product of these two sides multiplied by the cosine of the angle between them. This statement may be expressed in formula form as follows:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

For the triangle shown in figure 1-25, you know that side c measures 10.01 ft; side b, 12.00 ft; and angle A (included between them), 41°24'. The cosine of 41°24' is 0.75011. The solution for side a is as follows:

$$a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$a = \sqrt{144 + 100.20 - 2(12)(10.01)(0.75011)}$$

$$a = \sqrt{244.20 - 180.20}$$

$$a = \sqrt{64}$$

$$a = 8.00 \text{ ft}$$

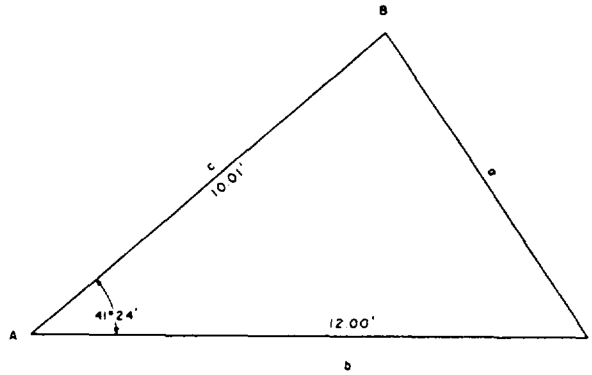

Figure 1-25.-Oblique triangle (law of cosines).

Knowing the length of this side, you can now solve for the remaining values by applying the law of sines.

If you know all three sides of a triangle, but none of the angles, you can determine the size of any angle by the law of cosines, using the following formulas:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

For the triangle shown in figure 1-26, you know all three sides but none of the angles. The solution for angle A is as follows:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{144 + 100.20 - 64}{2(12)(10.01)}$$

$$\cos A = \frac{180.20}{240.24}$$

$$\cos A = 0.75008$$

The angle with cosine 0.75008 measures (to the nearest minute) 41°24.

Solution by Law of Tangents

The law of tangents is expressed in words as follows: In any triangle the difference between two sides is to their sum as the tangent of half the difference of the opposite angles is to the tangent of half their sum.

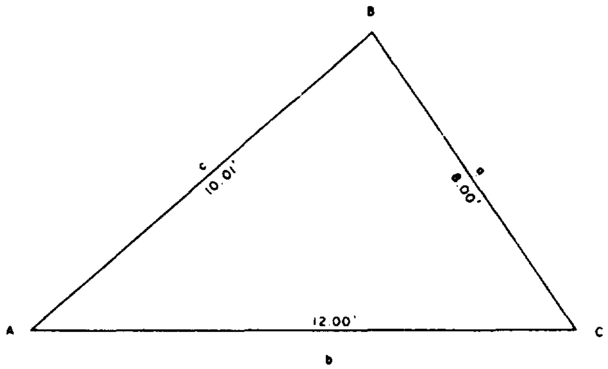

Figure 1-26.-Any triangle, three sides given.

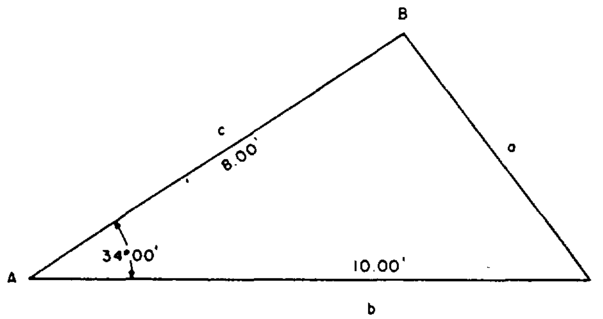

Figure 1-27.-0blique triangle (law of tangents).

For any pair of sides—as side a and side b—the law may be expressed as follows:

$$\frac{a - b}{a + b} = \frac{\tan 1/2\,(A - B)}{\tan 1/2\,(A + B)}$$

For the triangle shown in figure 1-27, you know the lengths of two sides and the size of the angle between them. You can determine the sizes of the other two angles by applying the law of tangents as follows.

First note that you can determine the value of angles (B + C), because (B + C) obviously equals 180° – A, or 180° – 34°, or 146°. Now, if you know the sum of two values and the difference between the same two, you can determine each of the values as follows:

$$\begin{aligned} x + y &= 5 \\ \underline{x - y} &\underline{= 1} \\ \text{(add) } 2x &= 6 \\ x &= 3 \\ y &= 5 - x \\ y &= 2 \end{aligned}$$

Now, you know the sum of (B + C). Therefore, if you could determine the difference, or (B – C), you could determine the sizes of B and C You can determine 12(B — C) from the law of tangents, written as follows:

$$\tan \frac{1}{2}(B - C) = \frac{(b - c)\tan \frac{1}{2}(B + C)}{b + c}$$

One-half of (B + C) means one-half of 146°, or 73°. The tangent of 730 is 3.27085. The solution for 12(B – C) is therefore as follows:

$$\tan \frac{1}{2}(B - C) = \frac{(10 - 8)(3.27085)}{10 + 8}$$

$$\tan \frac{1}{2}(B - C) = \frac{6.54170}{18} = 0.36342$$

(from table of natural tangents) 1/2 (B - C) = 19°58' (B – C) = 2(19058') = 39°56'

Knowing both the sum (B + C) and the difference (B – C), you can now determine the sizes of B and C as follows:

$$\begin{aligned} B + C &= 146°00' \\ \underline{B - C} &\underline{= \;\, 39°56'} \\ 2B &= 185°56' \\ B &= \;\, 92°58' \\ C &= (146° - 92°58') = 53°02' \end{aligned}$$

The Ambiguous Case

When the given data for a triangle consists of two sides and the angle opposite one of them, it may be the case that there are two triangles that conform to the data. A situation in which there can be two triangles is called the ambiguous case. Figure 1-28 shows two possible triangles that

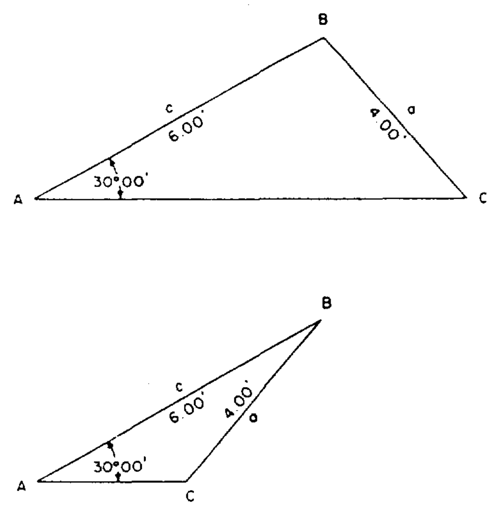

Figure 1-28.-Two ambiguous case triangles (solution of one will satisfy the other).

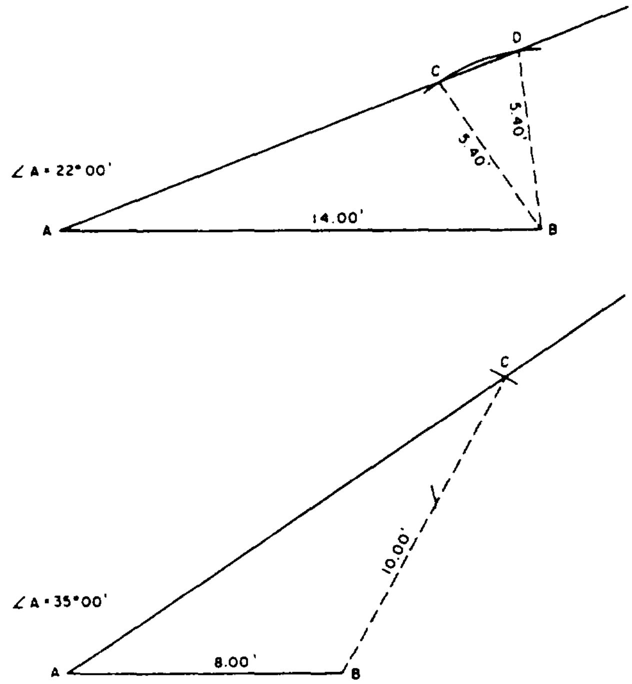

Figure 1-29.-Comparison of an ambiguous case triangle to a standard triangle.

might satisfy this situation. Both triangles shown are with given angle A = 30°00′, given side a = 4.00 ft, and given side c = 6.00 ft.

The best way to determine whether or not the given data for a triangle involves an ambiguous case is to lay out a figure to scale on the basis of the data, as shown in figure 1-29. Suppose, for example, that the data describes a triangle with angle A = 22°00'; side opposite, 5.40 ft; and other side, 14.00 ft. Lay off a line, AB, 14.00 ft long (to scale, of course), as shown in the upper triangle of figure 1-29. Use a protractor to lay off a line from A at 22°00'. Set a compass to the graphical distance of 5.40 ft (length of side opposite A) and with B as a center, strike an arc. You observe that this arc intersects the line from A at two places. Therefore, the triangle ACB and the triangle ADB both satisfy the data, and you have an ambiguous case.

Suppose now that the data describes a triangle with angle A = 35°00′; side opposite, 10.00 ft; and other side, 8.00 ft. Lay off the line AB 8.00 ft long as shown in the lower triangle of figure 1-29, and lay off a line from A at 35°00’, Set a compass to 10.00 ft (length of side opposite A) and with B as a center, strike an arc. This arc will intersect the line from A at only one point. Therefore, only one triangle satisfies the data.

Determination of Angle from Three Known Sides

There are several formulas for determining the size of an angle in a triangle from three known sides. The most convenient involves the versed sine of the angle, which means (1 -cos) of the angle. The formula goes as follows:

$$1 - \cos A = \frac{2(s-b)(s-c)}{bc}$$

$$1 - \cos B = \frac{2(s-a)(s-c)}{ac}$$

$$1 - \cos C = \frac{2(s-a)(s-b)}{ab}$$

The value s means one-half the sum of sides a, b, and c, or

$$s = \frac{a+b+c}{2}.$$

For the triangle shown in figure 1-30, you would determine the size of angle A as follows:

$$s = \frac{10.00 + 12.00 + 15.00}{2} = \frac{37.00}{2} = 18.50$$

$$1 - \cos A = \frac{2(18.50 - 15)(18.50 - 10)}{(15)(10)}$$

$$= \frac{2(3.50)(8.50)}{150}$$

$$1 - \cos A = \frac{59.50}{150} = 0.39667$$

$$\cos A = 1 - 0.39667 = 0.60333$$

The angle with cosine 0.60333 measures (to the nearest minute) 52°53′.

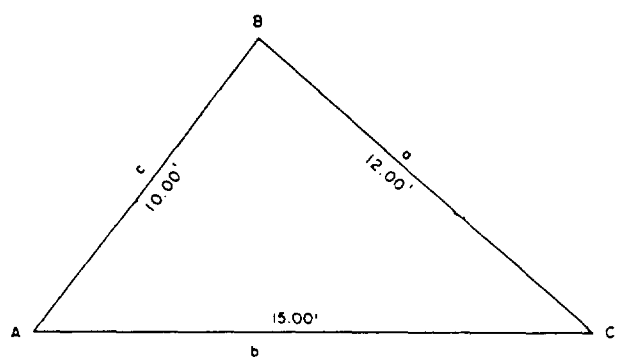

Figure 1-30.—Oblique triangle with three sides given and solved by versed sine formula.

Trigonometric Determination of Area

If you know all three sides of a triangle, you can determine the area by applying the following formula:

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where s = 1/2 perimeter of a triangle

For the triangle shown in figure 1-30, the area computation is

$$\text{area} = \sqrt{18.50(18.50 - 12.00)(18.50 - 15.00)(18.50 - 10.00}$$

$$\text{area} = \sqrt{18.50(6.50)(3.50)(8.50)}$$

$$\text{area} = \sqrt{3577.44}$$

$$\text{area} = 59.81 \text{ sq ft}$$

When you know two sides of a triangle and the angle included between them, you can determine the area by applying, appropriately, one of the following formulas:

$$\text{area} = 1/2bc \sin A$$

$$\text{area} = 1/2ac \sin B$$

$$\text{area} = 1/2ab \sin C$$

In figure 1-31, two sides, b = 13.00 ft and c = 9.00 ft, and the included angle, A = 40°00´, are given. The sine of 40°00´ is 0.64279. The area computation is as follows:

$$\text{area} = 1/2\ bc \sin A$$

$$\text{area} = 1/2(13.00)(9.00)(0.64279)$$

$$\text{area} = 58.50(0.64279)$$

$$\text{area} = 37.60 \text{ sq ft}$$

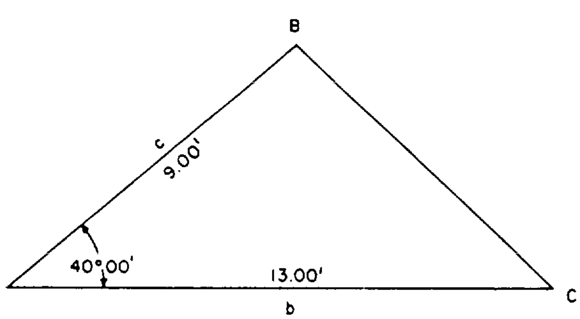

Figure 1-31.-Area of a triangle with two sides and one angle given.

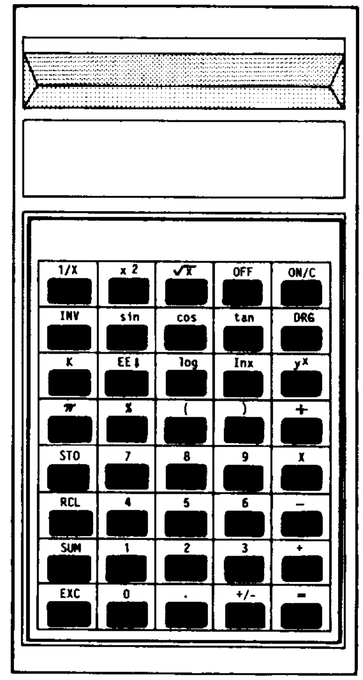

Figure 1-32.-Scientific pocket calculator.

SCIENTIFIC POCKET CALCULATOR

Figure 1-32 illustrates a typical pocket calculator that replaces the slide rule, logarithm tables, and office adding machine. This tool, packed with the latest in state-of-the-art solid-state technology, is a great asset for our trade. With it we can handle many problems quickly and accurately without having to hassle with lengthy, tedious computations. This tool should serve us faithfully for a long time if we treat it with respect and care.

Today's hand-held calculators have become an everyday part of our lives. Rugged and inexpensive, they're a practical answer to the real need we all have for quick, accurate calculations.

THE KEYBOARD

Your calculator has many features to make calculations easy and accurate. To allow you to use all of these features without crowding the keyboard, the designer has caused some of the keys to have more than one function. If you look closely at the keyboard, you'll notice that the keys in the column on the left side have two function symbols. These keys are called dual-function keys because they perform two functions. If you want to perform one of the first functions, simply press the key. To perform one of the second functions, you'll need to press the $\boxed{\text{2nd}}$ key and then press the key for the function you wish to perform.

INSTRUCTION MANUAL

Every calculator on the market should have an instruction manual enclosed with it. Check out all the features and functions summarized in the instruction manual to become familiar with what your calculator will (and will not) do for you.

HINTS ON COMPUTING

It is a general rule that when you are expressing dimensions, you express all dimensions with the same precision. Suppose, for example, you have a triangle with sides 15.75, 19.30, and 11.20 ft long. It would be incorrect to express these as 15.75, 19.3, and 11.2 ft, even though the numerical values of 19.3 and 11.2 are the same as those of 19.30 and 11.20.

It is another general rule that it is useless to work computations to a precision that is higher than that of the values applied in the computations. Suppose, for example, you are solving a right triangle for the length of side a, using the Pythagorean theorem. Side b is given as 16.5 ft, and side c, as 20.5 ft. By the theorem you know that side a equals the square root of $(20.5^2 - 16.5^2)$, or the square root of 148.0. You could carry the square root of 148.0 to a large number of decimal places. However, any number beyond two decimal places to the right would be useless, and the second number would be determined only for the purpose of rounding off the first.

The square root of 148.0, to two decimal places, is 12.16. As the 0.16 represents more than one-half of the difference between 0.10 and 0.20, you round off at 0.2, and call the length of side a 12.2 ft. If the hundredth digit had represented less than one-half of the difference between 0.10 and 0.20, you would have rounded off at the lower tenth digit, and called the length of side a 12.1 ft.

Suppose that the hundredth digit had represented one-half of the difference betwveen 0.10 and 0.20, as in 12.15. Some computers in a case of this kind always round off at the lower figure, as, 12.1. Others round off at the higher figure, as 12.2. Better balanced results are usually obtained by rounding off at the nearest even figure. By this rule, 12.25 would round off at 12.2, but 12.35 would round off at 12.4.

UNITS OF MEASUREMENT

Engineering science would not be so precise as it is today if it did not make use of systems of measurement. In fieldwork, drafting, office computation, scheduling, and quality control, it is important to be able to measure accurately the magnitudes of the various variables necessary for engineering computations, such as directions, distances, materials, work, passage of time, and many other things.

The art of measuring is fundamental in all fields of engineering and even in our daily lives. We are familiar, for instance, with "gallons," which determines the amount of gasoline we put in our car and with "miles," which tells us the distance we have to drive to and from work. It is also interesting to note that the development of most of these standard units of measure parallels the development of civilization itself, for there has always been a need for measurement. In the early days, people used night and day and the cycle of the four seasons as their measure of time. The units of linear measure were initially adopted as comparison to the dimensions of various parts of a man's body. For example, a "digit" was at that time the width of a man's middle finger, and a "palm" was the breadth of an open hand. The same applies to most other units of linear measure that we know today—like the "foot," the "pace," and the "fathom." The only difference between today's units of measure and those of olden days is that those of today are standardized. It is with the standard types of measurements that we are concerned in this training manual.

At present, two units of measurement are used throughout the world. They are the English system and the metric system, Many nations use the metric system.

The metric system is the most practical method of measurement, for it is based on the decimal system, in which units differ in size by multiples of tens, like the U.S. monetary system in which 10 mills equal 1 cent; 100 mills or 10 cents equal 1 dime; and 1,000 mills, 100 cents, or 10 dimes equal one dollar. When we perform computations with multiples of 10, it is convenient to use an exponential method of expression as you may recall from your study of mathematics.

Table 1-1.-Linear Conversion Factors

	Inches	Feet	Yards	Statute miles	Centimeters	Meters	Kilometers
Inch	1	.083333	.0277		2.540005	.0254	
Foot	12	1	.333		30.48006	0.304801	
Yard	36	3	1	.000568	91.44018	.914402	.000914
Statute mile	63,360	5280	1760	1		1609.347	1.609347
International nautical mile		6076.10	2025.36	1.150777			
United States nautical mile		6080.20	2026.73	1.151553			
Centimeter	.3937				1	.01	
Meter	39.37	3.280833	1.093611		100	1	.001
Kilometer				0.62137		1000	1
Decimeter	3.937	.328				.1	
Decameter	393.7	32.8				10	
Hectometer		328'-1"					.1
Myriameter				6.213712			10

A unit of measurement is simply an arbitrary length, area, or volume, generally adopted and agreed upon as a standard unit of measurement. The basic standard for linear measurement, for example, is the meter, and the actual length of a meter is, in the last analysis, equal to the length of a bar of metal called the International Meter Bar, one replica of which is kept in the National Bureau of Standards, Washington, D.C.

As an EA, you will not necessarily be working with all the units described in this chapter, and therefore need not attempt to memorize them all. Many are included simply to show that units are arbitrary and that there are many different kinds of units in use.

UNITS OF LINEAR MEASUREMENT

Linear measure is used to express distances and to indicate the differences in their elevations. The standard units of linear measure are the foot and the meter. In surveying operations, both of these standard units are frequently divided into tenths, hundredths, and thousandths for measurements. When longer distances are involved, the foot is expanded into a statute or to a nautical mile and the meter into a kilometer. Table 1-1 shows the conversion factors for the common linear measurements.

English Units

In the English system, the most commonly used basic unit of linear measurement is the foot, a unit that amounts to slightly more than three-tenths of the international meter. In what is called ENGINEER'S measurement, the foot is subdivided decimally; that is, into tenths, hundredths, or thousandths of a foot. In what is called CARPENTER'S measurement, or English units, the foot is subdivided into twelfths called inches, and the inch is further subdivided into even-denominator fractional parts, as 1/2 in., 1/4 in., 1/8 in., and so on.

Fractions or multiples of the basic 1-ft unit are used to form larger units of linear measure as follows:

1 link	=	0.66 ft
1 yard	=	3.00 ft
1 rod, pole, or perch	=	16.50 ft
1 Gunter's chain	=	66.00 ft
1 engineer's chain	=	100.00 ft
1 statute mile U.S.	=	5,280.00 ft
1 nautical mile (international)	=	6,076.10 ft

A unit of linear measurement, called a VARA of Spanish and Portuguese origin, was formerly used to measure land boundaries in those areas of the United States that were at one time under Spanish control. In those areas old deeds and other land instruments still contain property descriptions in varas, which vary from state to state and country to country from 32 to 43 in.

Metric Units

In many of the non-English-speaking countries of the world, the most commonly used basic unit of linear measure is the meter. The length of a meter was originally designed to equal (and does equal very nearly) one ten-millionth part of the distance, measured along a meridian, between the earth's equator and one of the poles. A meter equals slightly more than 1.09 yd.

The big advantage of the metric system is the fact that it is a decimal system throughout; that is, the fact that the basic unit can be both subdivided into smaller units decimally and converted to larger units decimally by simply moving the decimal point in the appropriate direction. Names of units smaller than the meter are indicated by the Latin prefixes deci- (one-tenth), centi- (one-hundredth), mini- (one-thousandth), and micro- (one-millionth), as follows:

$$1 \text{ decimeter} = 0.1 \text{ meter } (1 \times 10^{-1})$$

$$1 \text{ centimeter} = 0.01 \text{ meter } (1 \times 10^{-2})$$

$$1 \text{ millimeter} = 0.001 \text{ meter } (1 \times 10^{-3})$$

$$1 \text{ micrometer} = 0.000001 \text{ meter } (1 \times 10^{-6})$$

Names of units larger than the meter are indicated by the Greek prefixes deca- (ten), hecto- (one hundred), kilo- (one thousand), myria- (ten thousand), and mega- (one million), as follows:

$$1 \text{ decameter} = 10.00 \text{ meters } (1 \times 10)$$

$$1 \text{ hectometer} = 100.00 \text{ meters } (1 \times 10^{2})$$

$$1 \text{ kilometer} = 1{,}000.00 \text{ meters } (1 \times 10^{3})$$

$$1 \text{ myriameter} = 10{,}000.00 \text{ meters } (1 \times 10^{4})$$

$$1 \text{ megameter} = 1{,}000{,}000.00 \text{ meters } (1 \times 10^{6})$$

UNITS OF AREA MEASUREMENT

In the English and metric system, area is most frequently designated in units that consist of squares of linear units, as square inches, feet, yards, or miles; or square centimeters, meters, or kilometers. In the English system, the land-area measurements most commonly used are the square foot and the acre. Formerly the square rod (1 rod = 16.5 ft) and the square Gunter's chain (1 Gunters's chain = 66 ft) were used. One of the area measurements, with its equivalents, is as follows:

$$\begin{aligned} 1 \text{ acre} &= 10 \text{ sq Gunter's chains} \\ &= 160 \text{ sq rods} \\ &= 43{,}560 \text{ sq ft} \end{aligned}$$

An equilateral rectangular (square) acre measures 208.71 ft on a side. There are 640 acres in a square mile.

Other area equivalents that may be of value to you are as follows:

1 square inch (sq in.)	=	6.4516 + square centimeters (sq cm)
1 square foot (sq ft)	=	144 sq in.
	=	0.0929 + square meter (sq m)
1 square yard (sq yd)	=	9 sq ft
	=	0.8361 – sq m
1 square meter (sq m)	=	10.7639 sq ft
	=	1.1960 + sq yd

Actually, more attention should be given to linear equivalents. If you know the linear conversion factor from one unit to the other, you can always compute for any equivalent area or even volume. Just remember, area is expressed in square units and volume is expressed in cubic units.

Example: Find the area of a rectangle 2 ft by 3 ft in square inches.

Area = 2 ft x 3 ft = (2 x12)(3 x 12) = 864 sq in.

UNIT OF VOLUME MEASUREMENT

From your study of mathematics, you learned that volume is the measure of the amount of space that matter occupies. It is expressed in certain cubic units, depending upon the linear measurements or dimensions of the object.

As an EA, you will find that your interest in unit volume of measurements will be from the standpoint of earthwork, construction materials, material testing, rainfall runoff, and capacities of structures, such as, for example, a reservoir. The accuracy of your computations will depend upon your knowledge of the correct conversion factors and the units used. Remember that your dimensions must always be expressed in one kind of unit of measure; for instance, if you are using the meter, all dimensions must be in meters. The basic units of volume that you might be using are as follows:

1 cubic inch (cu in.) = 16.3872 cubic centimeters (cc)

1 cubic foot (cu ft) = 1,728 cu in. = 0.0283 cubic mete (cu m)

1 cubic yard (cu yd) = 27 cu ft = 0.7646 cubic meter

1 cubic meter (cu m) = 1,000,000 cc = 35.3145 cu ft

= 1.3079 cu yd

1 U.S. gallon = 231 cu in.

1 cu ft = 7.4805 gal

1 acre ft = 43,560 cu ft

= 1,233.49 cu m

UNITS OF WEIGHT

The units of weight most frequently used in the United States for weighing all commodities except precious stones, precious metals, and drugs are the units of the so-called AVOIRDUPOIS system. Avoirdupois units of weight are as follows:

437 1/2 grains (gr) = 1 ounce (oz)

16 ounces (oz) = 1 pound (lb)

100 pounds (lb) = 1 hundred-weight (c

1,000 pounds (lb) = 1 kip (K)

2,000 pounds (or 20 cwt) = 1 short ton (T)

2,200 pounds (lb) = 1 long ton

Precious stones and precious metals are usually weighed in the United States by the system of TROY weight, in which there are 12, rather than 16, oz in the pound. Drugs are weighed by APOTHECARIES' weight, in which there are also 12 oz in the pound.

The basic unit of the metric system of weight is the GRAM, which contains 15.432 grains. The GRAIN was originally supposed to be equal to the weight of a single grain of wheat, The gram of 15.432 grains is also used in the avoirdupois, troy, and apothecaries' system of weights.

Multiples and subdivisions of the basic unit of metric weight (the gram) are named according to the usual metric system of nomenclature, as follows:

0.000001 g = 1 microgram

0.001 g = 1 milligram

0.01 g = 1 centigram

0.10 g = 1 decigram

10.00 g = 1 decagram

100.00 g = 1 hectogram

1,000.00 g = 1 kilogram

A METRIC TON equals 1,000 kilograms, which equals 1.1 short tons.

The Engineering Aid is interested in the weight of his instruments and the pull to be applied to the ends of the tape to give correct linear measurements. The common units of weight in surveying are the OUNCE, the POUND, the GRAM, and the KILOGRAM. The following tabulation gives the relationship between these units:

1 ounce (oz) = 28.3495 grams (g)

1 pound (lb) = 453.5924 g = 0.4536 kg

1 kilogram (kg) = 2.2045 lb = 35.27 oz

UNITS OF ANGULAR MEASUREMENT

ANGULAR or CIRCULAR MEASURE is used for designating the value of horizontal and vertical angles. For general use in the measurement of angles, the circumference of the circle is divided into some even number of equal parts. The unit of angular measure is the angle at the center of the circle subtended by one of the small subdivisions of the circumference. The various units of angular measure are known as UNITS OF ARCS. In practice these units of arcs may be further expressed in decimal or fractional parts.

The Engineering Aid may encounter three systems of angular measure in the use of surveying instruments. They are the sexagesimal, the centisimal or metric, and the mil system.

Sexagesimal or North American System

In the sexagesimal or North American system, the circle is divided into 360 equal parts known as DEGREES of arc, each degree into 60 equal parts known as MINUTES of arc, and each minute into 60 equal parts known as SECONDS of arc. As an example, angles in this system are written as 263047'16".48 which is read as "two hundred sixty-three degrees, forty-seven minutes, and sixteen point four eight seconds of arc." In the United States, this is the most commonly used system of angular measurement.

Centisimal or Metric System

In the centisimal or metric system, the full circle is divided into four quadrants, and each quadrant is divided into 100 equal parts known as GRADS or GRADES. Each grad is further divided into decimal parts. As an example, angles in this system are written as $376^g.7289$, or $376^g72'89''$ which is read as "three hundred seventy-six point seven two eight nine grads," or as "three hundred seventy-six grads, seventy-two centisimal minutes, and eighty-nine centisimal seconds."

Mil System

In the mil system, the circle is divided into 6,400 equal parts known as MILS. The mil is divided into decimal parts. As an example, angles in this system are written as 1728.49 roils, which is read as "one thousand seven hundred twenty-eight point four nine mils." This system is used principally by the artillery people. The significance of this unit of angular measure is the fact that 1 mil is the angle that will subtend 1 yd at a range of 1,000 yd.

The relationship among values in the three systems of angular measure are as follows:

1 circle = 360 degrees = 400 grads = 6,400 mils

1 degree = 1.1111 grads = 17.7778 mils

1 minute = 0.2963 mils

1 grad = 0.9 degree = $0°54'00''$ = 16 mils

1 mil = 0.0562 degree = $0°03'22''.5$ or 3.3750 minutes

= 0.0625 grad

MORE UNITS OF MEASUREMENT

Aside from the units of measurement discussed above, the EA must also deal with other units of measurement, such as TIME, TEMPERATURE, PRESSURE, and so forth. He must use exact time in computing problems in astronomy and some laboratory works. He must be able to apply temperature corrections to his tape readings. He must also evaluate the effect of atmospheric pressure at different elevations and get involved in some other types of measure that will be discussed in the following paragraphs.

Time Measurement

For practical purposes in everyday affairs and in surveying, the measurement of time intervals is of great concern. The time used in everyday life is known as STANDARD TIME and is based on the mean apparent revolution of the sun around the earth because of the earth's rotation on its axis. Standard time is used in surveying to regulate the normal day's operations. But, when it is necessary to observe the sun or the stars to determine the azimuth of a line or the position of a point on the earth's surface, the surveyor uses three other kinds of time. They are APPARENT (true) SOLAR TIME, CIVIL (mean solar) TIME, and SIDEREAL (star) TIME. You will learn more about these different times when you study the chapter on "Geodesy and Field Astronomy" in *Engineering Aid 1 & C.*

In all four kinds of time, the basic units of measure are the YEAR, DAY, HOUR, MINUTE, and SECOND of time. The duration of any one of these units is not the same for all kinds of time. For example, the sidereal day is approximately 4 min shorter than a standard- or civil-time day.

In the practice of surveying, it is customary to say, or write, the time of day as the number of hours, minutes, and seconds since midnight. Then the recorded time would appear, for example, as $16^h37^m52^s.71$ which is read as "sixteen hours, thirty-seven minutes, and fifty-two point seven one seconds of time."

Units of time measure are sometimes used to designate the sizes of angles. The longitude of a point on the earth's surface is often expressed in this manner. The relationship between the units of time measure and the units of angular measure in the sexagesimal system are as follows:

$$1 \text{ hour} = 15 \text{ degrees } (1^h = 15°)$$

$$1 \text{ minute of time} = 15 \text{ minutes of arc } (1^m = 15')$$

$$1 \text{ second of time} = 15 \text{ seconds of arc } (1^s = 15'')$$

$$1 \text{ degree} = 4 \text{ minutes of time } (1° = 4^m)$$

$$1 \text{ minute of arc} = 4 \text{ seconds of time } (1' = 4^s)$$

$$1 \text{ second of arc} = 0.0667 \text{ second of time } (1'' = 0.0667^s$$

Temperature Measurement

In certain types of measurement, when the existing temperature differs from a standard temperature, the measured values will be in error and must be corrected, In each of the several temperature-measurement scales, the unit of measure is called a DEGREE, which varies for the different temperature scales, When the scale extends below zero, values below zero are identified by a minus sign. Temperatures are written, for example, as 23°F or – 5°C, the letter designating the particular temperature scale. To avoid confusion when writing or talking about temperature, we should always be sure to indicate the type of scale used, Two of the most commonly used temperature scales are the CENTIGRADE scale and the FAHRENHEIT scale.

On the Centigrade scale (also known internationally as "Celsius Scale" after Anders Celsius, a Swedish astronomer who first devised it), zero is the freezing point of water, and plus 100 is its boiling point.

On the Fahrenheit scale, the temperature of the freezing point of water is plus 32°, and its boiling point is plus 212°.

Now let us compare these scales. A Fahrenheit degree represents five-ninths of the change in heat intensity indicated by a degree on the Centigrade scale. Temperatures on either of the two scales can be converted to the other by the following formulas:

$$\text{Degrees C} = 5/9 \text{ (degrees F} - 32°,$$

$$\text{Degrees F} = (9/5 \text{ degrees C}) + 32°$$

Note that, when converting Fahrenheit to Centigrade, you should first subtract the 32°, then multiply by 5/9. When converting Centigrade to Fahrenheit, you should first multiply by 9/5, then add the 32°.

Pressure Measurement

Measurements of atmospheric pressure are used in surveying to determine approximate differences in elevation between points on the earth's surface and to determine the best approximate correction for the effect of atmospheric refraction. The units of measure for atmospheric pressure and their relationships are as follows:

1 atmosphere = 29.9212 inches of mercury

= 760 millimeters of mercury

= 14.6960 pounds per square inch

= 1,03323 kilograms per square centimeter

= 33.899 feet of water

= 1.01325 bars, or 1013.25 millibars

Dry Measure

Dry measure is a system of measure of volume used in the United States for dry commodities, such as grains, fruits, and certain vegetables. The basic unit in dry measure is the BUSHEL. The standard U.S. bushel contains about 77.6 lb of water. Since there are about 62.4 lb of water in a cu ft, it follows that a U.S. bushel has a volume of

$$\frac{77.6}{62.442}, \text{ or about } 1\ 1/4 \text{ cu ft.}$$

Units of dry measure are as follows:

1 bushel = 4 pecks

1 peck = 8 quarts

1 quart = 2 pints

Board Measure

Board measure is a method of measuring lumber in which the basic unit is a BOARD FOOT (bf). A board foot is an abstract volume 1 ft long by 1 ft wide by 1 inch thick. The chief practical use of board measure is in cost calculations; lumber is sold by the board foot just as sugar is sold by the pound.

There are several formulas for calculating the number of board feet in any given length of lumber of given section dimensions. Because lumber dimensions are most frequently given by length in feet and width and thickness in inches, the following formula is probably the most practical:

$$bf = \frac{\text{thickness in in.} \times \text{width in in.} \times \text{length in ft}}{12}$$

Board measure is calculated on the basis of the nominal, not the actual, section dimensions. The actual section dimensions of (for example) 2 by 4 stock, which is surfaced on all four surfaces (S4S), are about 1 5/8 in. thick by 3 5/8 in. wide. Nevertheless, the computation for the number of (for example) 300 linear ft of 2 by 4 stock would be as follows:

$$\frac{\overset{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times \overset{100}{\cancel{300}}}{\underset{1}{\underset{\cancel{3}}{\underset{\cancel{6}}{\cancel{12}}}}} = 200 \text{ bf}$$

Liquid Measure

In the United States the basic unit of liquid measure is the GALLON, which has a volume of 231 cu in. or 0.13 cu ft. The gallon is subdivided into smaller units as follows:

1 gallon = 4 quarts

1 quart = 2 pints

1 pint = 4 gills

Units larger than the gallon in liquid measure are as follows:

1 barrel = 31.5 gallons

1 hogshead = 63 gallons or 2 barrels

For petroleum products the standard barrel contains 42 gallons.

In the metric system the basic unit of liquid measure is the LITER, equal in volume to a cubic decimeter, or about 61 cu in. There are 3.785 liters in a U.S. gallon.

Following the usual metric system of nomenclature, subdivisions and multiples of the liter are as follows:

0.000001 liter = 1 microliter

0.001 liter = 1 milliliter

0.01 liter = 1 centiliter

0.10 liter = 1 deciliter

10.00 liter = 1 decaliter

100.00 liter = 1 hectoliter

1,000.00 liter = 1 kiloliter

Electrical Measure

In an electrical circuit there is a flow of electrons, roughly similar to the flow of water in a water pipe. The flow is occasioned by the production, at a generating station, battery, or other source, of an ELECTROMOTIVE FORCE (E), roughly similar to the "head" of water in a water system. The size of the electromotive force is measured in units called VOLTS.

The rate of flow of the electrons through the circuit is called the CURRENT (I). Current is measured in units called AMPERES.

The usual conductor for transporting a flow of electrons through a circuit is wire. Generally speaking, the smaller the diameter of the wire, the more will be the RESISTANCE (R) to the flow, and the larger the diameter, the less the resistance. Resistance is measured in units called OHMS.

The definitions of the units volt, ampere, and ohm are as follows:

1 volt	Electromotive force required to send a current of 1 ampere through a system in which the resistance measures 1 ohm.
1 ampere	Rate of flow of electrons in a system in which the electromotive force is 1 volt and the resistance, 1 ohm.
1 ohm	Resistance offered by a system in which the electromotive force is 1 volt and the current, 1 ampere.

The ohm is named for Georg Simon Ohm, a German scientist and early electrical pioneer, who discovered that there is a constant relationship between the electromotive force (E), the current (I), and the resistance (R) in any electrical circuit. This relationship is expressed in “Ohm’s law” as follows:

$$I = \frac{E}{R}$$

From the basic law it follows that

$$E = IR$$

$$R = \frac{E}{I}$$

From Ohm’s law you can (1) determine any one of the three values when you know the other two and (2) determine what happens in the circuit when a value is varied.

Suppose, for example, that the resistance (R) is increased, while the electromotive force (E) remains the same. It is obvious that the current (I) must drop proportionately. To avoid a drop in the current, it would be necessary to increase the electromotive force proportionately.

When an electrical circuit is open (that is, when there is a break in the circuit, such as an open switch), there is no flow of electrons through the circuit. When the circuit is closed, however, the current will begin to flow. With a constant electromotive force (E), the rate at which the current (1) flows will depend on the size of the resistance (R). The size of the resistance will increase with the number of electrical devices (such as lights, motors, and the like) that are placed on the circuit, and the amount of POWER each of these consumes.

Power may be defined as “electrical work per unit of time. ” James Watt, another early pioneer in the electrical field, discovered that there is a constant relationship between the electromotive force (E), the current (I), and the power consumption (P) in a circuit. This relationship is expressed in the formula P = IE, from which it follows that

$$I = \frac{P}{E}, \text{ and } E = \frac{P}{I}.$$

Power is measured in units called WATTS, a watt being defined as the work done in 1 second when 1 ampere flows under an electromotive force of 1 volt.

Suppose, now, that you have a 110-volt circuit in your home. The constant E of this circuit, then, is 110 volts. In the circuit there is probably a 15-ampere fuse. A fuse is a device that will open the circuit by “burning out” if the current in the circuit exceeds 15 amperes. The reason for the existence of the fuse is the fact that the wiring in the circuit is designed to stand safely a maximum current of 15 amperes. A current in excess of this amount would cause the wiring to become red hot, eventually to “burn out, ” and perhaps to start an electrical fire.

Suppose you light a 60-watt bulb on this circuit. Your E is 110 volts. By the formula

$$I = \frac{P}{E},$$

you know that the current in the circuit with the 60-watt bulb on is

$$\frac{60}{110},$$

or about 0.54 amperes, which is well within the margin of safety of 15 amperes. Dividing 15 amperes by 0.54 amperes you find that this fuse will protect a 27-lamp circuit.

But suppose now that you place on the same one-lamp circuit an electric toaster taking about 1,500 watts (electrical devices are usually marked with the number of watts they consume) and an electrical clothes dryer taking about 1,200 watts. The total P is now 60 + 1,500 + 1,200, or 2,760 watts. The current will now be

$$\frac{2{,}760}{110}$$

or 25 amperes. Theoretically, before it reaches this point, the 15-ampere fuse will burn out and open the circuit.

Mechanical Power Measure

Mechanical power (such as that supplied by a bulldozer) is measured in units called FOOT-POUNDS PER SECOND (ft-lb/sec) or FOOT-POUND PER MINUTE (ft-lb/min). A foot-pound is the amount of energy required to raise 1 lb a distance of 1 ft against the force of gravity.

One HORSEPOWER equals 33,000 ft-lb/sec or 550 ft-lb/min. One horsepower equals about 746 watts.

CONVERSION OF UNITS

To convert a measure expressed in terms of one unit to the equivalent in terms of a different unit is, when you know the ratio between the units, a simple proportional equation problem. Suppose, for example, that you want to convert a linear distance in engineer's measure (feet and decimals of feet) to the equivalent in carpenter's measure (feet and twelfths of feet) to the nearest one-eighth in. Suppose that the original distance is 12.65 ft. This means "12 ft and 65 hundredths of a foot." You want to determine first, then, how many twelfths of a foot there are in 65 hundredths of a foot. The original ratio is 12/100. The proportional equation solution is as follows:

$$\frac{x}{65} = \frac{12}{100}$$

$$x = \frac{12 \times 65}{100} = \frac{780}{100} = 7.8$$

Therefore, there are 7.8 in. (twelfths of a foot) in 0.65 ft. The next step is to determine how many eighths of an in. there are in 0.8 in.; that is, in eight-tenths of an in. The initial ratio is 8/10, and the proportional equation solution is as follows:

$$\frac{x}{8} = \frac{8}{10}$$

$$x = \frac{8 \times 8}{10} = \frac{64}{10} = 6.4$$

Therefore, there are (rounded off) 6/8 in,, or 3/4 in., in 0.8 in. In 12.65 ft, then, there are 12 ft 7 3/4 in. to the nearest 1/8 in.

Actually, the proportional method used above can be simplified by using the following solution:

Convert 12.65 ft to the nearest 1/8 in. in carpenter's measure.

12.65 ft = 12 ft + (0.65 x 12 = 7.8 in,)

= 12 ft 7.8 in.

= 12 ft 7.0 in. + (0.8 x 8 = 6.4 eighths)

= 12 ft 7.0 in. + 6/8 in. or 3/4 in. to the nearest eighth in.

= 12 ft 7 3/4 in.

In converting from engineer's to carpenter's linear measure, or vice versa, surveyors working with values to only the nearest 0.01 ft frequently use the following conversions to decimal equivalents of inches from 1 through 11 and decimal equivalents of the common carpenter's-measure subdivisions of the inch.

1 in. = 0.08 ft

2 in. = 0.17 ft

3 in. = 0.25 ft

4 in. = 0.33 ft

5 in. = 0.42 ft

6 in. = 0.50 ft

7 in. = 0.58 ft

8 in. = 0.67 ft

9 in. = 0.75 ft

10 in. = 0.83 ft

11 in, = 0.92 ft

$\frac{1}{8}$ in. = 0.01 ft

$\frac{1}{4}$ in. = 0.02 ft

$\frac{1}{2}$ in. = 0.04 ft

$\frac{3}{4}$ in. = 0.06 ft

Using these values, you can convert decimals of a foot to inches carpenter's measure, or inches carpenter's measure to decimals of a foot, very easily. To convert (for example) 0.37 ft to inches carpenter's measure, you have the following:

$$\begin{aligned} 0.33 \text{ ft} &= 4 \text{ in.} \\ 0.04 \text{ ft} &= \tfrac{1}{2} \text{ in.} \\ \hline 0.37 \text{ ft} &= 4\tfrac{1}{2} \text{ in.} \end{aligned}$$

To convert (for example) 7 3/8 in. carpenter's measure to engineer's measure, you have the following:

$$7 \text{ in.} = \qquad 0.58 \text{ ft}$$

$$\frac{3}{8} \text{ in.} = (3 \times 0.01) = \underline{0.03} \text{ ft}$$

$$7\frac{3}{8} \text{ in.} = \qquad 0.61 \text{ ft}$$

For a great many types of conversions there are tables in which you can find the desired values by inspection. Various publications contain tables for making the following conversions:

Meters to feet

Feet to meters

Degrees Centigrade to degrees Fahrenheit

Degrees Fahrenheit to degrees Centigrade

Inches and sixteenths to decimals of a foot

Sixteenths of an inch to decimals of a foot

Minutes to decimals of a degree

Degrees to roils and roils to degrees

Grads to degrees, minutes, and seconds

A conversion factor is a number that, if multiplied by a value expressed in terms of one unit, will produce the equivalent value expressed in terms of a different unit. The factor for converting linear feet to miles, for instance, is 0.00019. If you multiply 5,280 ft by 0.00019, you get 1.0032 miles, which is close enough to a mile to satisfy most practical purposes.

When you know the ratio between two different units, you can easily work out your conversion factor. For example, you know that the ratio of degrees to roils is

$$\frac{9}{160}.$$

The conversiom factor for converting degrees to roils is the number of roils in 1 degree, which is

$$\frac{160}{9}, \text{ or } 17.8.$$

The conversion factor converting roils to degrees is the number of degrees in a roil, which is

$$\frac{9}{160}, \text{ or } 0.0562.$$

Some of the common conversion factors are as follows :

Linear feet x 0.00019 = miles

Linear yards x 0.0006 = miles

Square inches x 0.007 = square feet

Square feet x 0.111 = square yards

Square yards x 0.0002067 = acres

Acres x 4840.0 = square yards

Cubic inches x 0.00058 = cubic feet

Cubic feet x 0.03704 = cubic yards